PMAT 501/601 L01 Winter 2009
Assignment 1

Questions taken from the text by D. Cohn will be specified by page and number. Due Feb. 4, 2009.

1. In the category $\mathcal{S}$ of sets and functions, show
   (a) $f : A \to B$ has a left (post) inverse iff f is injective,
   (b) $g : B \to C$ has a right (pre) inverse iff g is surjective.

2. Prove that a linearly ordered set $(X, <)$ has the property that every non-empty subset $A \subseteq X$ with an upper bound has a least upper bound, iff it has the property that every non-empty subset $B \subseteq X$ with a lower bound has a greatest lower bound.

3. Find the cardinality of the set of all continuous functions $\mathbb{R} \to \mathbb{R}$.

   [Hint : Explain why the restriction map $\hom_{Top}(\mathbb{R}, \mathbb{R}) \to \hom_{Top}(\mathbb{Q}, \mathbb{R})$ is injective, then follow this by the inclusion map $\hom_{Top}(\mathbb{Q}, \mathbb{R}) \hookrightarrow \mathbb{R}^{\mathbb{Q}}$.]

4. p.7 - 1

5. p.7 - 4

6. p.7 - 5

7. p.7 - 8 [It will suffice to solve this question with $\mathbb{N}$ replaced by any finite set X, the question for $\mathbb{N}$ appears much more difficult.]