

PMAT 501/601 L01

Winter 2009 Assignment 5

Questions taken from the text by D. Cohn will be specified by page and number. Due April 6 , 2009.

1. Determine

$$\int_I f d\lambda ,$$

where f is the Cantor singular function. Also indicate why this function should be integrable.

2. p.69, 3

3. p.69, 4

4. p.69, 5

5. Let $f_n : X \rightarrow \mathbb{R}$ be any measurable functions, and set $B := \{x : \{f_n(x)\} \text{ is a Cauchy sequence}\}$. Show that B and B^c are in the σ -algebra \mathcal{A} .

Hint: It is probably easier to work with B^c and define, for each $k, N \in \mathbb{N}$, $C_{N,k} := \{x : \exists m, n \geq N \text{ with } |f_m(x) - f_n(x)| > 1/k\}$.

6. p.73, 1

7. p.73, 2