

PMAT 607 ASSIGNMENT 3 Due March 7, 2005

1. (a) Show that the space  $\mathbb{R}$  is contractible<sub>\*</sub> with basepoint 0.  
(b) Prove or disprove the same statement for  $\mathbb{Q}$ . [20]
2. Munkres, p.341-3 : If the base space  $X$  of a covering map  $Y \rightarrow X$  is connected, then all fibres  $F_x$  have the same cardinality . [20]  
[Hint: A function  $f : W \rightarrow Z$  is *locally constant* iff each  $w \in W$  has an open neighbourhood  $U$  such that  $f|_U$  is constant. Show that any locally constant function is continuous.]
3. Munkres, p.341-4 : The composition  $p = r \circ q$  of two covering maps  $r, q$  is again a covering map if all fibres of  $r$  are finite. [20]
4. Munkres p.341-5 : Show that the map  $p : S^1 \rightarrow S^1$  given by  $p(z) = z^n$  is a covering map. [20]
5. (part of Munkres, p.341-6) Given a covering map  $Y \rightarrow X$ , show [20]
  - (a) If  $X$  is  $T_2$  then so is  $Y$ ,
  - (b) If  $X$  is regular then so is  $Y$ ,
  - (c) If  $X$  is completely regular then so is  $Y$ .