PMAT 607 ASSIGNMENT 5 Due March 21, 2005

1. Munkres, p.356-1 : Given the polynomial $f(z) = z^n + a_{n-1}z^{n-1} + ... + a_0$, with $|a_{n-1}| + ... + |a_0| < 1$. Show that all zeros of f lie in the interior of the unit ball B^2. [20]

   [Hint: Let $g(z) = 1 + a_{n-1}z + ... + a_1 z^{n-1} + a_0 z^n$ and show that $g(z) \neq 0,\ z \in B^2$]

2. Find a possible source of error in Munkres' proof of the Fundamental Theorem of Algebra. [20]

3. Munkres, p.366-2 [20]

4. (a) Munkres p.341-6 : Given a covering map $p : Y \twoheadrightarrow X$ with X compact and $p^{-1}(x)$ finite for all $x \in X$, show that Y is compact. [20]

   (b) State and prove the converse of Question 4(a), with the additional assumption that Y is T_1.

5. Show that a T_2 space with infinite cardinality admits an infinite collection of non-empty mutually disjoint open subsets. [20]

   [Hint : It's easy to get two mutually disjoint non-empty open subsets A, B. Now consider two cases. The first case is when A (or B) is infinite. The second case is when both A, B are finite, in this case consider the complement of $A \cup B$.]