PMAT 607   ASSIGNMENT 6   Due April 1, 2005

1. (a)   Let $\mathcal{C}$ be a category in which all hom-sets are non-empty, i.e. for any objects $A, B \in \mathcal{C}$, $hom_{\mathcal{C}}(A, B) \neq \emptyset$. Supposing the product $A \prod B$ exists, show that the projection $\pi_1 : A \prod B \to A$ is an epimorphism, and similarly for π_2.

   (b)   State the dual result for coproducts. [20]

2. (a)   Show that for m, n relatively prime, $\mathbb{Z}_m \times \mathbb{Z}_n \approx \mathbb{Z}_{mn}$.

   (b)   Also show this is false if m, n are not relatively prime. [20]

3. Prove that any group is a quotient of a free group. [20]

4. Consider the subspaces of the unit interval $X := \{0\} \cup \{1/n : n \geq 1\}$, $X_k := \{0\} \cup \{1/n : n \geq k\}$ (note $X = X_1$). Now let $Z_k := X_1 \bigsqcup X_k$, and $f : Z_k \to X$ be the identity map on X_1 and the inclusion on X_k. Show that f is a covering projection. [20]

5. Give an example to show that the composition $p = r \circ q$ of two covering projections is not in general a covering projection. [20]

   [Hint : Let $r : Y = X \times \mathbb{N} \to X$ be the projection π_1 onto the first factor, and X the same space as in Question 4.]

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