PMAT 607 ASSIGNMENT 7 Due April 11, 2005

1. (a) Given a category $\mathcal{C}$ with a terminal object T, show that products in $\mathcal{C}$ are a special case of pull-backs.

   (b) State the dual result for coproducts. [20]

2. Given the push-out diagram below in the category $\mathcal{G}$ of groups : [20]

$$\begin{array}{ccc} K & \xrightarrow{\alpha} & G \\ \beta\big\downarrow & & \downarrow f \\ H & \xrightarrow{g} & P \end{array},$$

   show that if α is an epimorphism then so is g.

   [Hint : There are two ways to do this question, the easy way and the hard way.]

3. Show the following two groups are isomorphic : [20]

$$G = \langle a, b \,|\, a^2 b^2 \rangle \,, \qquad H := \langle x, y \,|\, xyx^{-1}y \rangle \ .$$

4. Give a cutting and pasting argument to show that the real projective plane $\mathbb{R}P^2$ is the union of a disc and a Möbius band, glued together by identifying their bounding circles. [20]

5. Similarly to Question 4, show the Klein bottle is the union of two Möbius bands with their bounding circles identified. [20]