PMAT 607 L01 W 2010

ASSIGNMENT 1 Due January 29

1. In the category $\mathcal{F}$ of fields and unit preserving homomorphisms, show that every morphism is a monomorphism. [15]

2. Give an example of a category in which every morphism is an epimorphism. [15]

3. (a) In $\mathcal{V}_k$, the category of vector spaces over a field k and linear transformations, show that a morphism is a monomorphism iff it is injective.

   (b) Show that a morphism in the category $\mathcal{V}_k$ is an epimorphism iff it is surjective. [15]

4. Show that whenever a composition of morphisms gf (in any category) is a monomorphism, then f is necessarily a monomorphism. State and prove a similar result for epimorphisms. [10]

5. A subspace A of a topological space X is called klopen iff $A = C \cap U$ for some closed set C and open set U. Show that A is klopen iff $\overline{A} \setminus A$ is closed. [15]

6. A subspace $A \subseteq X$ is called a retract of X if there is a map $r : X \to A$ such that $ri = 1_A$, where $i : A \hookrightarrow X$ is the inclusion map. Show that $\{0, 1\}$ is not a retract of $I = [0, 1]$. [10]

7. Let $(X, \leq)$ be a poset and $\mathcal{C}_X$ the corresponding category. [10]
   (a) What can you say about initial and terminal objects in $\mathcal{C}_X$?
   (b) What can you say about monomorphisms and epimorphisms in $\mathcal{C}_X$?

8. Classify the letters of the alphabet, as appearing on a standard English keyboard, up to homeomorphism. [10]