PMAT 607 L01 W 2010

ASSIGNMENT 2 Due Feb 23

1. (a) Every subspace of a T_2 (Hausdorff) space is T_2. [20]

   (b) Every closed subspace of a normal space is normal (normal means T_2 and disjoint closed subsets can be separated by disjoint open sets containing them).

   (c) Is any subspace of a normal space normal? Answer just "yes" or "no," if your answer is "no" then mention (without proof) a counterexample. [Hint : the answer can be found in Munkres.]

2. Let $\mathcal{S}, \mathcal{T}$ denote the categories of sets and topological spaces, respectively. Also, let $\mathbf{c}$ denote the cardinality of the continuum. [15]

   (a) Show that $|hom_{\mathcal{S}}(\mathbb{R}, \mathbb{R})| = 2^{\mathbf{c}}$.

   (b) Show that $|hom_{\mathcal{T}}(\mathbb{R}, \mathbb{R})| = \mathbf{c}$.

3. If A is a retract of a T_2 space $X \supseteq A$ then A is closed in X. [15]

4. Give an example to show that the hypothesis of normality is necessary in the Tietze Extension Theorem. [Hint : An example exists using a space X consisting of only 3 points.] [15]

5. Consider the group $G = \langle x, y \mid xyx = yxy \rangle$. Show that G is non-abelian and that it is infinite. [15]

6. Munkres, p.330-3 [20]