PMAT 607 L01 W 2010

ASSIGNMENT 3

Due March 16 in Lecture

1. (a) Show that a contractible space is path connected.

   (b) Prove or disprove the statement : $\mathbb{Q}$ is contractible. [20]

2. Munkres, p.334 - 1. A subset $A \subseteq \mathbb{R}^n$ is said to be *star convex* if for some point $a_0 \in A$, all the line segments joining a_0 to other points of A lie in A.
   (a) Find a star convex set that is not convex (a sketch will suffice).
   (b) Show that if A is star convex then A is contractible.
   (c) Show that a star convex set is simply connected. [20]

3. Munkres, p.186 11(b). Let $p : X \twoheadrightarrow Y$, $q : W \twoheadrightarrow Z$ be identification maps. Also suppose that X, Z (or W, Y) are both locally compact Hausdorff. Show that the map $p \times q : X \times W \twoheadrightarrow Y \times Z$ is also an identification map. [20]

4. Munkres, p.335 - 4. Suppose $A \subseteq X$ and $r : X \twoheadrightarrow A$ is a retraction. Show that

   $$r_* : \pi_1(X; a_0) \rightarrow \pi_1(A; a_0)$$

   is surjective. [20]

5. Let $\mathcal{C}$ be the category of left R-modules, where R is a ring with unity $1 \neq 0$. Show that for any two objects $A, B \in \mathrm{Obj}(\mathcal{C})$, the coproduct and product and direct sum are all isomorphic, i.e. [20]

   $$A \coprod B \approx A \prod B \approx A \oplus B.$$