PMAT 607 L01 W 2010

ASSIGNMENT 4

Due April 1 in Lecture

1. Give an example of a surjective map $p : Y \twoheadrightarrow X$ whose fibres are all discrete and all have the same cardinality, but is not a covering projection. [18]

2. Definition 3.2.1 in Selick, *Introduction to Homotopy Theory*, reads as follows. A map $p : E \to X$ is called a covering projection if every point $x \in X$ has an open neighbourhood U_x such that $p^{-1}(U_x)$ is a disjoint union of open sets each of which is homeomorphic to U_x. This "definition" does not imply that p is surjective, but even assuming p is surjective give an example of such a map that is *not* a covering projection. [18]

3. Show that a T_2 space with infinite cardinality admits an infinite collection of non-empty mutually disjoint open subsets. [Hint: It's easy to get two mutually disjoint non-empty open subsets A, B. Now consider two cases. The first case is when A (or B) is infinite. The second case is when both are finite, in this case consider the complement of $A \cup B$. [16]

4. Munkres, p.366-2. For each of the following spaces the fundamental group is either trivial, $\mathbb{Z}$, or the free group F_2 on two generators (which is the fundamental group of $S^1 \bigvee S^1$). Give π_1 for each and explain your reasoning. [12]
   (a) The solid torus $B^2 \times S^1$,
   (b) The torus T^2 with a single point removed,
   (c) The cylinder $S^1 \times I$,
   (d) The infinite cylinder $S^1 \times \mathbb{R}$
   (e) $\mathbb{R}^3$ with the non-negative x, y, z axes removed,
   (f) $\{x : \|x\| > 1\} \subset \mathbb{R}^2$.

5. Munkres, p.341-4. The composition $p = r \circ q$ of two covering projections is again a covering projection if all fibres of r are finite. [18]

6. (a) Munkres p.341-6. Given a covering projection $p : Y \twoheadrightarrow X$ with X compact and $p^{-1}(x)$ finite for all $x \in X$, show that Y is compact.

   (b) Also show that if Y is compact and X is T_1, then every fibre $p^{-1}(x)$ must be finite. [18]