PMAT 607 L01 W 2010

MIDTERM

Due March 23 in Lecture

1. Give an example of an open cover of the reals $\mathbb{R}$ which has no Lebesgue number. Then generalize to an open cover of $\mathbb{R}^n$ which has no Lebesgue number. [20]

2. (Munkres, p.144 - 2 = (a) and (b))

   (a) Any morphism in $\mathcal{T}op$ that has a right (pre) inverse is a proclusion.

   (b) Any retraction map r is a proclusion.

   (c) Let $A \subseteq X$. Show that the quotient space X/A is T_1 if and only if A is closed.

   (d) If X is T_3 and A is closed, show X/A is T_2. [20]

3. Show that the map $\kappa : S^n \twoheadrightarrow \mathbb{R}P^n$ given by $\kappa(x) = \kappa(-x)$ is a covering projection. [20]

4. Consider a map $f : X \to X$ of a space X to itself. One defines $x \in X$ to be a fixed point of f if $f(x) = x$. Let x, y be fixed points of f, and suppose γ is a path in X from x to y.

   (a) Show that $f\gamma$ is also a path from x to y.

   (b) Define $x \sim y$ if and only if there exists a path γ from x to y such that $\gamma \simeq f\gamma$ rel$\{0, 1\}$. Show that this is an equivalence relation on Fix(f), the set of fixed points of f. [20]

5. (part of Munkres, p.341-5) Given a covering projection $p : Y \twoheadrightarrow X$, show [20]

   (a) if X is T_1 then so is Y,

   (b) if X is T_2 then so is Y,

   (c) if X is T_3 then so is Y,

   (d) if X is completely regular then so is Y.