PMAT 613 L01 Fall 2009
Assignment 4

Due Oct 27, 2009.

1. For any Galois connexion $\star : \mathcal{F} \to \mathcal{G}$ and $\dagger : \mathcal{G} \to \mathcal{F}$, show $X^{\star\dagger\star} = X^{\star}$, $X \in \mathcal{F}$ and $Y^{\dagger\star\dagger} = Y^{\dagger}$, $Y \in \mathcal{G}$.

2. Given field extensions $L : M : K$ with $L : K$ a splitting field for $f(t)$ over K, show that $L : M$ is a splitting field for $f(t)$ over M.

3. Show that there is a euclidean construction for trisecting the angle $2\pi/5$.

4. Show that the euclidean construction of a regular 9-gon is impossible.

5. If $L : K$ is a splitting field for $f(t)$ over K with $\deg(f(t)) = n$, then $[L : K] \mid n!$. [See text p. 114 for a hint].

6. Show the Galois groups $\Gamma(\mathbb{R}, \mathbb{Q})$ and $\Gamma(\mathcal{A}_r, \mathbb{Q})$ are both trivial, where $\mathcal{A}_r$ denotes the real algebraic numbers. [Hint : use the fact that $\mathbb{R}$ is ordered, as well as any of its subfields, and show automorphisms will preserve positivity here.]

7. If K is a finite field and $f(t) \in K[t]$ is irreducible, show that $f(t)$ is separable. [Hint : contradiction and 13.13(b).]

8. True or False, with counterexamples if false.

   (a) If S is a finite set, $S \subseteq T$, and $|S| = |T|$, then $S = T$.

   (b) The same is true for infinite sets.

   (c) There is only one field homomorphism $\mathbb{Q} \to \mathbb{Q}$.

   (d) If K, L are subfields of $\mathbb{C}$, then there exists at least one field homomorphism $K \to L$.

   (e) Same question as (d) with the additional hypothesis $[K : \mathbb{Q}] = [L : \mathbb{Q}]$.

(f) Distinct automorphisms of a field K are linearly independent over K.

(g) Linearly independent field monomorphisms are distinct.