PMAT 613 L01 Fall 2009
Assignment 5

Due December 8, 2009, in lecture.

1. Determine each of the following Galois groups.
   (a) $\Gamma(\mathbb{Q}(\sqrt{2}, \sqrt{5}), \mathbb{Q})$,
   (b) $\Gamma(\mathbb{Q}(\omega), \mathbb{Q})$, where $\omega^p = 1$, $\omega \neq 1$, and p is prime.
   (c) $\Gamma(K, \mathbb{Q})$, where K is a splitting field over $\mathbb{Q}$ for $f(t) = t^4 - 3t^2 + 4$.

2. Determine the Galois group of any quadratic $f(t) = t^2 + at + b$, $a, b \in \mathbb{Q}$, over $\mathbb{Q}$. [Hint : There are two possible answers.]

3. Determine the Galois group of any polynomial $f(t) \in \mathbb{C}[t]$ over $\mathbb{C}$. [Hint : There is only one possible answer.]

4. The Thompson-Feit theorem is one of the most celebrated of the twentieth century. It says that any finite group of odd order is solvable. Explain, in one or two sentences, what this implies when applied to Galois theory.

5. For question 1(a) above, indicate all normal subgroups H of the Galois group, and what $H^\dagger$ is in each case.