PMAT 613   L01   Fall 2009
Assignment 3

Questions taken from the text by I. Stewart (3rd Edition) will be specified by page and number. Due Oct 27, 2009.

1. Apples sell for 61 cents each and grapefruits sell for 74 cents each. Jane goes to the store and buys some apples and some grapefruits, spending $30.00 total. How many of each did she purchase? Use the euclidean algorithm and Z-process (or equivalent).

2. Show that the concept of an algebraic simple extension is well defined. Thus, if $L = K(x)$ is a simple algebraic extension, and also $L = K(y)$, you must show that y is an algebraic element of L.

3. Show that the units of a (not necessarily commutative) ring form a group under multiplication. What does this imply for K^* when K is a field?

4. (a) For a PID D, show that p is prime iff $p|(ab)$ implies $p|a$ or $p|b$, assuming as usual p is not 0 or a unit. [Half of this result was already done in the lectures]. This second concept is often called irreducible.

   (b) Give an example of a domain D where prime and irreducible are not equivalent.

5. In $\mathbb{Z}[t_1, t_2, t_3]$, write
   (a) $t_1^3 + t_2^3 + t_3^3$ as a polynomial in $\sigma_1, \sigma_2, \sigma_3$,
   (b) do the same for $t_1^2 t_2 + t_2^2 t_1 + \ldots + t_3^2 t_2$.

6. 4.2 For $X \subset \mathbb{C}$, find $\overline{X}$, the subfield generated by X, in each of the following cases.
   (a) $X = \{0, 1\}$,
   (b) $X = \{0\}$,
   (c) $X = \{0, 1, i\}$,
   (d) $X = \{i, \sqrt{2}\}$,

(e) $X = \{\sqrt{2}, \sqrt{3}\}$,

(f) $X = \mathbb{R}$,

(g) $X = \mathbb{R} \cup \{i\}$.

7. (a) 4.4   Prove that $1, \sqrt{2}, \sqrt{3}, \sqrt{6}$ are linearly independent over $\mathbb{Q}$ [the text gives hints].

(b) Determine whether $L : \mathbb{Q}$ is a simple extension, where $L = \mathbb{Q}(\sqrt{2}, \sqrt{3})$.

8. 4.10, giving counterexamples when false.