UNIVERSITY OF CALGARY

FACULTY OF SCIENCE

Mathematics and Statistics - MS 476
Ph: (403) 220-5203

COURSE INFORMATION SHEET

WINTER 2010

1. **Course**: PMAT 685 (Galois Representations)
   Lecture/Time: L01
   Instructor: Clifton Cunningham
   Office/Phone/Email: MS528/220-6888/cunning@math.ucalgary.ca

2. **Prerequisites**: PMAT 613 or permission from the Department
   NOTE: The Faculty of Science policy on pre- and co-requisite checking is outlined in the current University Calendar (see www.ucalgary.ca/pubs/calendar) *Faculty of Science, section 5C*. **It is the students' responsibility to ensure that they have the pre- and co-requisites for the course, and if they do not they will be withdrawn from the course without notice.**

3. **Fee policy:** After the last day to drop/add courses, there will be no refund of tuition fees if a student withdraws from a course, courses or the session.

4. **Academic Accommodations**: It is the student's responsibility to request academic accommodations. A student with a documented disability who may require academic accommodation must register with the Disability Resource Centre to be eligible for formal academic accommodation. DRC registered students are required to discuss their needs with the instructor no later than fourteen (14) days after the start of this course.

5. **The University policy on grading and related matters** is described in the current University Calendar, *Academic Standings*. In determining the overall grade in the course, the following weights will be used:

   Exercises [approx. 20] %100

   As the course progresses, each student will contribute to the course notes by solving roughly 20 exercises assigned during the lectures. Solutions will be edited discussed in class. The veracity, accuracy, thoroughness and elegance of these contributions will determine the fina grade awarded in this course. More detail about the course is available from the course website at http://blackboard.ucalgary.ca.

6. **Missed Components of Term Work.** The regulations of the Faculty of Science pertaining to this matter are outlined in the current University Calendar, *Faculty of Science, section 6A*. It is the student's responsibility to be familiar with these regulations.

7. **Academic misconduct** (cheating, plagiarism, or any other form) is a very serious offence that will be dealt with rigorously in all cases. A single offence may lead to disciplinary probation or suspension or expulsion. The Faculty of Science follows a zero tolerance policy regarding dishonesty. Please read the sections of the current University Calendar. See: http://www.ucalgary.ca/honesty/

8. **Dates and times of class exercises held outside of class hours (evening tests, Saturday laboratory examinations, weekend field trips, etc.):**

   *****THERE WILL BE NO OUT-OF-CLASS-TIME ACTIVITY.*****

REGULARLY SCHEDULED CLASSES HAVE PRECEDENCE OVER ANY OUT-OF-CLASS-TIME ACTIVITY. If you have a conflict with any out of class time activity, please inform your instructor at least one week in advance of the activity so that other arrangements may be made for you.

______________________________ Date: _______________
Associate Dean's approval for out of regular class time activity

______________________________ Date: _______________
Department approval

Overview

Although this course is called Galois Representations, a more accurate title would be *Complex and ℓ-adic Representations of Local and Global Weil-Deligne Groups and Associated L-functions*. Since every (complex or ℓ-adic) representation of a Galois group of a local or global field gives rise to a (complex or ℓ-adic) representation of a Weil-Deligne group, the scope of the course is such that it does indeed include the study of Galois representations.

We will begin by making a brief study a Galois representations of global fields. Our guide here will be [Diamond-Shurman 2007, Chapter 9]:

9: Galois Representations

- 9.1 Galois number fields
- 9.2 The ℓ-adic integers and the ℓ-adic numbers
- 9.3 Galois representations
- 9.4 Galois representations and elliptic curves
- 9.5 Galois representations and modular forms
- 9.6 Galois representations and Modularity

After reading sections 9.1, 9.2 and 9.3, we will begin our study of representations of local Weil-Deligne groups, especially those arising from elliptic curves, following [Rohrlich 1994]. A copy of this paper is posted on the course website at `https://blackboard.ucalgary.ca`. The Table of Contents of this paper is:

Part I: The Weil-Deligne Group

1. The Weil group
2. Representations of the Weil group
3. The Weil-Deligne group and its representations
4. The Weil-Deligne group and ell-adic representations
5. Indecomposable representations and special representations
6. A second point of view on the Weil-Deligne group
7. Invariant forms
8. The L-factor

9. L-factors and ell-adic representations
10. The conductor
11. The epsilon factor
12. The root number

Part II: Elliptic Curves

13. The representation associated to an elliptic curve
14. The case of potential good reduction
15. The case of potential multiplicative reduction
16. The Weil pairing
17. The L-factor
18. The conductor
19. The root number
20. The Archimedean case
21. The global case

In order to read this paper carefully it will be necessary to fill in some gaps in your background. We will do this as we go and each of you will make some such contributions to the course notes.

Representations of local Weil-Deligne groups arise from a number of other sources besides elliptic curves, including admissible representations according to the Local Langlands Conjecture. While the machinery that produces a representation of a local Weil-Deligne group from an admissible representation is in general opaque, to say the least, certain cases are well understood. If time permits, we will discuss some examples of the Local Langlands Conjecture, and explain its place in the Langlands Programme.

After we've finished with [Rohrlich 1994], we return to [Diamond-Shurman 2007] again, paying closer attention to Sections 9.4, 9.5 and 9.6. Galois representations arise from modular forms and, more generally, automorphic representations. Almost certainly, time will not permit us to treat this topic with the respect it deserves, but we will do our best.

In the public eye, the most exciting development in the field of Galois representations is the proof of Fermat's Last Theorem. While we will certainly talk about this proof, it will not be the heart of the course. For a course on Galois Representations along these lines I recommend [Goins], which is based on [].

As the course proceeds, each of you will contribute to the course notes in the form of lecture notes and/or exercises; the veracity, accuracy, thoroughness and elegance of your contribution will determine your final grade in the course.