

## Lab 4

REVIEW ALL of chapter 3 and 4 (up to, and including Bayes' Theorem) for the next quiz! Do all the questions that pertain to these sections.

1. A coin with is tossed randomly on to the plane  $\mathbb{R}^2$ . Find the probability that the coin does not intersect any line of the form  $x = k$  where  $k$  is an integer and where
  - (a) the diameter of the coin is  $\frac{1}{2}$ .
  - (b) The diameter of the coin is  $\frac{1}{4}$ .
2. Let  $i$  represent the sum of the points in two tosses of a die. Find the probability distribution of  $i$ . Which sum occurs most frequently?
3. In the above problem, let  $Y$  denote the larger of the two numbers appearing on the two dice. Determine the probability distribution of  $Y$
4. A hunter finds a crow sitting in a tree. He has only 3 shells in his shotgun. Assuming that the crow does not fly away after it is shot at, and the probability that the hunter hits the crow on each shot is 0.4, find the probability distribution of the number of shots fired at the crow.
5. Suppose that  $X$  takes on one of three values, 1, 2, or 3. If  $P(X=1) = 0.4$ , and  $P(X=2) = 0.1$ , what is the  $P(X=3)$  if  $X$  has a probability distribution.
6. A contractor will bid for 2 jobs in sequence. She has a probability of 0.5 of winning the first job. If she wins the first job, then she has a 0.2 chance of winning the second job: if she loses the first job, then she has a 0.4 chance of winning the second job. Let  $X$  denote the number of jobs she wins. Find the probability distribution of  $X$
7. Suppose  $A$  and  $B$  are events such that  $P(A) = 0.25$ ,  $P(B) = 0.40$ , and  $P(A \cup B) = 0.50$ 
  - (a) Determine
    - (i)  $P(A \cap B)$
    - (ii)  $P(A^c \cap B^c)$
    - (ii)  $P(A^c)$
    - (iii)  $P(B^c)$
  - (b) Are  $A$  and  $B$  disjoint? Independent? Explain using probabilities
3. 80% of the patrons at hockey games buy a soft drink. 60% buy popcorn and 50% purchase both. What is the probability that a person:
  - (a) doesn't buy a soft drink or doesn't buy popcorn?
  - (b) buys only a soft drink?
4. A stock is selected at random each morning from the Toronto Stock Exchange. It is observed that there is a 50% chance that a stock will go up that day, a 10% chance that it is worth more than \$100 and a 45% chance that it will go down and be worth \$100 or less.
  - (a) What is the probability that the stock goes up and is worth more than \$100?
  - (b) What is the probability that the stock goes down and is worth more than \$100?

5. A card is drawn from a well shuffled deck of 52 cards. Let  
 A = card chosen is King or Queen  
 B = card chosen is a picture card (J, Q, K)  
 C = card is a club  
 D = card is a king or a spade
- Determine whether or not the following pairs of events are independent: (Hint: use conditional probabilities)
- (A,B)
  - (A,C)
  - (B,D)
6. An automatic drilling machine produces good pieces 91%. Faulty pieces are drilled either off-centre (C) or at an incorrect angle (A). In the total production, 5% are drilled off-centre and 2% are drilled both off-centre and at an incorrect angle.
- What percentage is drilled at an incorrect angle?
  - If a part is drilled off-centre, what is the probability that it will be drilled off-angle?
  - Is “off-centre” drilling independent of “off-angle” drilling? Explain
7. A certain company encourages its employees to participate in some type of physical activity. A survey revealed that 40% play golf, 50% fish and 25% play golf and fish. Define the events as  
 A: play golf                      B: fishes
- $P(A^c \cup B)$
  - $P(A \cap B^c)$
  - $P(A \cup B)$
  - $P(A^c \cap B^c)$
8. The failure rate of a heart attack alarm in an ICU is 0.001. For improved safety, a duplicate alarm is installed. What is the probability that a heart attack will not be signaled if the alarms work independently of one another?
9. A house is randomly chosen in Calgary. The probability is 0.35 that the house is occupied by a single parent and 0.03 that it is occupied by an elderly person (who is not a single parent – not that they couldn't be.)
- What is the probability the house chosen does not have a single parent as a resident?
  - What is the probability the house chosen is occupied by single parent or an elderly person?
  - What is the probability the house chosen does not have a single parent nor an elderly person living as a resident?

10. 50 people at a conference were classified by town of residence and sex, giving the following results:

Sex	City			Total
	Calgary (C1)	Red Deer (C2)	Edmonton (C3)	
Male (A1)	8	9	13	30
Female (A2)	2	6	12	20
Total	10	15	25	50

A person is selected at random from those attending the conference.

- What is the probability that the person selected is from Calgary?
  - What is the probability that the person selected is not from Red Deer?
  - What percentage of the participants are women?
  - What is the probability that the person chosen is from Calgary and Red Deer?
  - What is the probability that the person chosen is male and from Red Deer?
  - What is the probability that the person chosen is from Red Deer or Edmonton?
  - What is the probability that the person chosen is female or is from Edmonton?
  - What is the probability that the person chosen is male if the person selected is from Edmonton?
  - What is the probability that the person chosen is from Red Deer given that a male is selected?
  - Are the events “Male” and “from Calgary” independent? Disjoint events? Explain using probability values from this question.
11. Imagine that you achieved a perfect score on last week’s midterm examination. Once word got out, three of your friends and relatives sent you congratulatory fruit baskets. After opening the baskets and inspecting all of the lovely fruit you begin to feel a mite peckish. You select a basket at random, and then randomly select a piece of fruit from the basket selected. The first basket contained 4 apples, 4 oranges, and 4 bananas; the second basket contained 4 apples, 5 oranges, and 6 bananas; the third contained 1 pineapple and six oranges.
- If the piece of fruit you selected was an apple, then what is the probability that you selected basket 1?
  - If the piece of fruit you selected was an orange, then what is the probability that you selected basket 3?
  - If the piece of fruit you selected was a pineapple, then what is the probability that you selected basket 3?
12. Each computer chip produced by machine A is defective with a probability of 0.1, whereas each computer chip produced by machine B is good with a probability of 0.95. 42% of computer chips are produced by machine A, the remainder by machine B. A chip is chosen at random.
- What is the probability that the chip is defective?
  - Given that the chip is defective, what is the probability the chip was produced by machine A?