Lab 5

1. Let X represent the sum of the points in two tosses of a die.
   (a) Find the probability distribution of X.
   (b) Find the mean and variance of X.

2. In the above problem, let Y denote the smaller of the two numbers appearing on the two dice.
   (a) Find the probability distribution of Y.
   (b) Find the mean and standard deviation of Y.

3. A hunter finds a crow sitting in a tree. He has only 3 shells in his shotgun. Assuming that the crow does not fly away after it is shot at, and the probability that the hunter hits the crow on each shot is 0.4.
   (a) Find the probability distribution of X.
   (b) Find the mean and variance of X.

4. Suppose that X takes on one of three values, 1, 2, or 3. If P(X=1) = 0.4, and P(X=2) = 0.1, what is the P(X=3)? Find the mean and variance of X.

5. A contractor will bid for 2 jobs in sequence. She has a probability of 0.5 of winning the first job. If she wins the first job, then she has a 0.2 chance of winning the second job: if she loses the first job, then she has a 0.4 chance of winning the second job. Let X denote the number of jobs she wins.
   (a) Find the probability distribution of X.
   (b) Find the expected number of jobs she wins and its variance.

6. An urn contains 8 white balls and 4 red balls. Three balls are selected at random from the urn.
   (a) Find the probability distribution of the number of red balls chosen in the sample.
   (b) Find the mean and standard deviation of the probability distribution in (a).
   (c) If you were given $2 for every red ball chosen, how much money can you expect to make?
   (d) If you have to pay $1 to play and you are given $1 for each red ball chosen, how much money are you expected to make?

7. A game of chance is considered **fair** if a player's expectation is equal to zero. If someone pays me $10 each time I roll a 3 or 4 with a balanced die, how much should I pay them when I roll a 1,2,5, or 6 to make the game fair?

8. *Simplified Craps*. A bet of $1 is taken to play the game. Two fair dice are rolled, and sum of the points on the dice is recorded. If the sum is 12, you win $5; if it is 2 you win $3; if a 10 or 11 appears you win $2, and if a 3 or a 4 appears you win $1. Any other outcome results in the loss of your initial wager. Should you play this game?

9. Three balanced coins are tossed independently. One of the variables of interest is X, the number of heads. Let Y denote the amount of money won on a side bet (after you pay a dollar to play) in the following manner. If the first head occurs on the first toss, you win $1. If the first head occurs on toss 2 or on toss 3 you win $2 or $3 respective. If no heads appear, you don't get anything (you win -$1).
   (a) Find the joint probability distribution for X and Y.
   (b) Find the expected amount that you would win. Should you make a side bet?

10. A local supermarket has three checkout counters. Two customers arrive at the counters at different times when the counters are serving no other customers. Each customer chooses a counter at random, independently of the other. Let X denote the number of customers who choose counter 1 and Y, the number who select counter 2.
    (a) Find the joint probability distribution for X and Y.
    (b) Find the cov(X,Y) and ρ(X,Y).
    (c) Find the var(X+Y)
    (d) Are the random variables X and Y independent?

11. From a group of three Liberals, two Conservatives, and one independent, a committee of two people is to be randomly selected. Let X denote the number of Liberals and Y, the number of Conservatives on the committee.
  (a) Find the joint probability distribution for X and Y.
  (b) Find cov(X,Y) and ρ(X,Y) and interpret ρ(X,Y).
  (c) Find the var(X+Y).
  (d) Are the random variables X and Y independent?

12. The following is a joint probability distribution for X and Y.

		X		
Y		1	2	
	0	.38	.17	.55
	1	.14	.02	.16
	2	.24	.05	.29
		.76	.24	1

  (a) Find cov(X,Y) and ρ(X,Y).
  (b) Find the var(X+Y).
  (c) Are the random variables X and Y independent?
  (d) (i) Find the marginal distribution for Y
   (ii) Find the probability distribution for Y = X²+2X+3 and the mean of Y where x = 0,1,2 using the same probabilities as the marginal distribution for Y

13. Find the mean and variance of X
$$f(x) = \begin{cases} 1 - x/2, \, 0 \le x \le 2 \\ 0, \qquad elsewhere \end{cases}$$

14. The temperature X at which a thermostatically controlled switch turns on has probability density function given by
$$f(x) = \begin{cases} 1/2, \quad 59 \le x \le 61 \\ 0, \qquad elsewhere \end{cases}$$
  (a) Find E(X) and var(X)
  (b) Find F(X)
  (c) Find an interval shorter than (59,61) in which at least ¾ of the measurements must lie.

15. A sample of 100 values has a mean of 41 and a standard deviation of 6. The shape of the distribution is unknown. How many values would you expect to be contained in the interval 20 to 62?

16. For a list of 50 sample values, the mean is 1 and the standard deviation is 3.5. In what interval would you expect to find at least 93% of the data?