

Statistics 213 Assignment 3

1. Each computer chip produced by machine A is defective with a probability of 0.1, whereas each computer chip produced by machine B is good with a probability of 0.95. 42% of computer chips are produced by machine A, the remainder by machine B. A chip is chosen at random.
 - (a) What is the probability that the chip is defective? (0.071)
 - (b) Given that the chip is defective, what is the probability the chip was produced by machine A? (0.5915)

2. Imagine that you achieved a perfect score on last week's midterm examination. Once word got out, three of your friends and relatives sent you congratulatory fruit baskets. After opening the baskets and inspecting all of the lovely fruit you begin to feel a mite peckish. You select a basket at random, and then randomly select a piece of fruit from the basket selected. The first basket contained 4 apples, 4 oranges, and 4 bananas; the second basket contained 4 apples, 5 oranges, and 6 bananas; the third contained 1 pineapple and six oranges.
 - (a) If the piece of fruit you selected was an apple, then what is the probability that you selected basket 1? (5/9)
 - (b) If the piece of fruit you selected was an orange, then what is the probability that you selected basket 3? (9/16)
 - (c) If the piece of fruit you selected was a pineapple, then what is the probability that you selected basket 3? (1)

3. Tom is playing a game on the Price is Right. He is given tags with prices on them. In order to win an item, he must place the correct tag on the item.
 - (a) How many ways can he arrange the 4 tags if there are 4 items? (24)
 - (b) If he is given 5 tags, how many ways can he arrange the tags when there are 4 items? (120)
 - (c) If he is given 5 tags and there are only 3 items, how many ways can he arrange the tags? (60)

4. A meeting is held with 8 individuals. If each person shakes hands with another person only once, how many handshakes occurred at the meeting? (28)

5. A woman has 11 close friends.
 - (a) Find the number of ways she can invite 5 of them to dinner. (462)
 - (b) Find the number of ways she can invite 5 of them to dinner, but where 2 of the friends are married and will not attend separately. (210)
 - (c) Find the number of ways she can invite 5 of them to dinner, but where 2 of the friends are not on speaking terms and will not attend together and where
 - a. she wants to invite one of them. (252)
 - b. doesn't care if one or both do not attend. (378)

6. A student must answer 10 out of 13 questions.
 - (a) How many choices are there? (286)
 - (b) How many if the student must answer the first 2 questions? (165)
 - (c) How many if the student must answer the first or second question but not both? (110)

7. A meeting is held at a round table with 7 people, how many different ways can they be arranged? (720)

8. Let the random variable X represent the sum of the points in two tosses of a die.
 - (a) Find the probability distribution of X. Which sum occurs most frequently?
 - (b) Find the expected sum on a roll of the dice. (7)
 - (c) Find the standard deviation on a roll of the dice. (2.415)

9. In the above problem, let Y denote the smaller of the two numbers appearing on the two dice. Determine the probability distribution of Y , as well as the mean of Y and the standard deviation of Y . (2.5278) (1.4040)
10. An urn contains 8 white balls and 4 red balls. Three balls are selected at random from the urn.
- Find the probability distribution of the number of red balls chosen in the sample. Based on the probability distribution, what is the probability that less than 2 red balls will be chosen out of the urn? (0.7636)
 - Find the mean and standard deviation of the probability distribution in (a). (1.000) , (0.7385)
11. A hunter finds a crow sitting in a tree. He has only 3 shells in his shotgun. Assuming that the crow does not fly away after it is shot at, and the probability that the hunter hits the crow on each shot is 0.4, find the probability distribution of the number of shots fired at the crow. In addition, find the expected number of shots fired at the crow as well as the variance of the number of shots fired at the crow. (1.96), (0.7584)
12. Suppose that X is a random variable that takes on one of three values, 1, 2, or 3. If $P(X=1) = 0.4$, and $P(X=2) = 0.1$, what is the $P(X=3)$? (.5)
13. A contractor will bid for 2 jobs in sequence. She has a probability of 0.5 of winning the first job. If she wins the first job, then she has a 0.2 chance of winning the second job; if she loses the first job, then she has a 0.4 chance of winning the second job. Let X denote the number of jobs she wins. Find the probability distribution of X , as well as expected value ($E(X)$) and variance ($VAR(X)$). (0.8), (.36)
14. A casino owner decided to charge \$2 to play the following game. 3 cards are drawn from a deck of 52. If no hearts are drawn the player gets nothing; if 1 heart is drawn the player wins \$1; if 2 hearts are drawn the player wins \$5. How high should the owner make the jackpot on a pull of 3 hearts if he wants the "house" to average 50 cents a play? (\$29.05)
15. *Simplified Craps*. A bet of \$1 is taken to play the game. Two fair dice are rolled, and sum of the points on the dice is recorded. If the sum is 12, you win \$5; if it is 2 you win \$3; if a 10 or 11 appears you win \$2, and if a 3 or a 4 appears you win \$1. Any other outcome results in the loss of your initial wager. Should you play this game? (-0.36) No
16. A greedy hunter shoots at a mallard duck sitting on a lake. The probability that the hunter hits the duck is $1/3$. If the duck is missed, it is equally likely to fly away as it is to stay on the lake. The hunter has 3 shells and he shoots until the duck is hit or it flies away. Find the expected number of shots fired and the standard deviation of the number of shots fired. (1.4444), (0.6850)
17. A die is tossed, and if a 1 or 2 appears then 3 balls are taken from a bag containing 3 red and 3 blue. Otherwise the same number of balls are taken from a different bag containing 1 red and 5 blue. If X represents the number of red balls chosen in this experiment, find $E(X)$ and $Var(X)$ (0.8333), (0.5389)
18. The probability that Ms. Brown will sell a piece for property at a profit of \$3000 is $3/20$, a profit of \$1500 is $7/20$, break even is $7/20$, and the probability that she will lose \$1500 is $3/20$. What is her expected profit? (\$750)
19. A game of chance is considered **fair** if a player's expectation is equal to zero. If someone pays me \$10 each time I roll a 3 or 4 with a balanced die, how much should I pay them when I roll a 1,2,5, or 6 to make the game fair? (\$5)

Minitab instructions

Click on **Start** in the lower left-hand corner of the screen. Click **MathStats Packages** and then **Minitab 14 for Windows**. Finally, click on **Minitab**.

You are now in the program. Our main interest will be in the DATA window, which serves as the worksheet for entering data, and the MENU bar that allows the selection of commands to perform various tasks. The SESSION window permits entry of typed commands that duplicate the aforementioned procedures ... and so will not be used initially. However, results will be presented in this window.

1. The probability that a person who undergoes a kidney operation will recover is 0.6. Find the probability that of 5 patients who undergo similar operations,
(a) none will recover (b) not more than one will recover (c) at least 3 will recover

From the MENU BAR select CALC>PROBABILITY DISTRIBUTIONS>BINOMIAL..... a dialog box appears. There are various ways in which you can find the required answers

- ii. If a single probability is needed as in part a) of the example:
Select PROBABILITY from the options listed (clicking on the circle, enters a dot.)
In the box by NUMBER OF TRIALS, type the number 5.
In PROBABILITY OF SUCCESS, insert 0.6.
As INPUT CONSTANT, specify the "x" value from the question (0 for part a)). Click on OK
The probability then comes up on the SEESION screen. [0.0102]
- iii. When a sum of probabilities is involved, as in part b), begin in the same way {CALC>PROB DISTR>BINOM}
Select CUMULATIVE PROBABILITY from the options.
Enter NUMBER OF TRIALS and PROBABILITY OF SUCCESS (5 and 0.6) as before.
As INPUT CONSTANT, specify the "x" from the question (1 for part b)). Click on OK
The number appearing on the SESSION screen will be the $P(X \leq x)$. { $P(X \leq 1)$ for b)}
[0.08704]

This method could also be used for part c), but the required probabilities would have to be expressed in terms of the cumulative probability and the complement..... $P(X \geq 3) = 1 - P(X \leq 2)$ [0.68256]

- iv. When several different questions are asked about the same distribution, it would be helpful to have all of the probabilities for each individual "x" calculated at once.
Enter the values 0,1,2,3,4,5 into c1, the first column of the worksheet.
Then proceed as in the previous examples:
From the menu bar, select CALC>PROBABILITY DISTRIBUTIONS>BINOMIAL;
select PROBABILITY from options; enter NUMBER OF TRIALS and PROBABILITY OF SUCCESS (5 and 0.6).
In the box associated with INPUT COMUMN indicate C1 (where you have listed the possible values of X) and request OPTIONAL STORAGE in C2. CLICK on OK>
On the worksheet, the probabilities for each value of X appear in column C2.

Repeating the procedure with CUMULATIVE PROBABILITY and choosing OPTIONAL STORAGE in C3 will complete a table of x values, individual probabilities, and cumulative probabilities in the WORKSHEET.

2. In Calgary, incompatibility is given as a legal reason for divorce in 70% of all divorce cases.
(a) What is the probability that in the next 6 divorce cases filed in Calgary, 5 will be due to incompatibility? (0.3025)
(b) If 5000 divorces occur each year in Calgary, what is the expected number of divorce cases due to incompatibility? (3500)
3. An automobile safety engineer figures that 1 in 10 automobile accidents are due to driver fatigue. What is the probability that at least 3 out of 5 accidents are due to driver fatigue? (0.00856)

4. Based on past experience, the probability that a student will pass this course is 0.7. From a random sample of 8 students,
- What is the probability that all 8 students pass this course? (0.0576)
 - What is the probability that at least 2 students pass this course? (0.9987)
 - In a class size of 107, how many students are expected to fail this course? (32.1)
5. The captain of a Navy gunboat orders a volley of 26 missiles to be fired at random along a 500ft stretch of shoreline that he hopes to establish as a beachhead. Dug into the beach is a 30 foot-long bunker serving as the enemy's first line of defense. What is the probability that exactly
- 3 missiles will hit the bunker? (0.1353)
 - Between 6 and 9 missiles, inclusive, will hit the bunker? (0.00376)
 - If at least 10 successful missiles are needed to destroy the bunker, what is the probability that the captain is unsuccessful in destroying the bunker? (1.00)
 - What is the expected number of missiles to hit the bunker? (1.56)

Minitab instructions

The Poisson instructions are similar to the binomial.

6. The number of mistakes in one page of a solutions manual to a statistics textbook follows a Poisson distribution with a rate of 2.2 mistakes per page.
- Find the probability that a randomly chosen page contains exactly 3 mistakes.
 - Find the probability that a randomly chosen page contains at most 4 mistakes.
 - Find the probability that a randomly chosen page contains at least 7 mistakes.

From the MENU BAR select CALC>PROBABILITY DISTRIBUTIONS>POISSON..... a dialog box appears. There are various ways in which you can find the required answers

- If a single probability is needed as in part a) of the example:
 Select PROBABILITY from the options listed (clicking on the circle, enters a dot.)
 In the box by MEAN, type 2.2
 As INPUT CONSTANT, specify the "x" value from the question (3 for part a)). Click on OK
 The probability then comes up on the SEESION screen. [0.1966]
- When a sum of probabilities is involved, as in part b), begin in the same way {CALC>PROB DISTR>POISSON}
 Select CUMULATIVE PROBABILITY from the options.
 Enter MEAN 2.2.
 As INPUT CONSTANT, specify the "x" from the question (4 for part b)). Click on OK
 The number appearing on the SESSION screen will be the $P(X \leq x)$. { $P(X \leq 4)$ for b)}
 [0.9275]

This method could also be used for part c), but the required probabilities would have to be expressed in terms of the cumulative probability and the complement..... $P(X \geq 7) = 1 - P(X \leq 6)$ [0.0075]

7. The number of cars arriving at the service station just down the street from my place follows a Poisson distribution with a rate of 12 per hour. Find the probability that within the next hour
- at most 10 cars will arrive. (0.3472)
 - exactly 15 will arrive. (0.0724)
 - At least 2 will arrive.(0.9999)

8. A data processing company uses 200 personal computers. The probability that any one of them will break on a given day is 0.01. There are 3 spare computers available and broken machines are always fixed overnight. Using the Poisson distribution, find the probability that:
- (a) on a given day, all computers will be used.(0.1804)
 - (b) On given day the number of spare computers is sufficient to replace all the broken ones. (0.8571)
9. A medical drop-in clinic receives, on average, 5 patients per hour. Find the probabilities of the following events:
- (a) at least 2 patients will arrive in a one hour period? (.9595)
 - (b) 9 patients will arrive in 90 minute period? (0.1144)
10. Telephone calls come through an exchange at a rate of 2.6 call per 10 minute interval. What is the probability that
- (a) exactly 2 calls in a **5 minute** interval? (0.2303)
 - (b) at least 2 calls in a 10 minute interval? (0.7326)
 - (c) no more than 1 call in a 1 minute interval? (.9715)
11. The number of mistakes in one page of a solutions manual to a statistics textbook follows a Poisson distribution with a rate of 2.2 mistakes per page. Find the probability that a randomly chosen page contains at most 4 mistakes.(.9271)

Additional questions.

12. The World Series is a best of 7 game series (whoever wins 4 games first will win the series) between Team A and Team B. The probability that Team A wins any game is 0.6. There are no ties in the World Series. Assuming the outcome of each game is independent of what has happened in prior games.
- (a) What is the probability that Team A wins the World Series? (0.7102)
 - (b) Given that Team A has won the first two games, what is the probability that the World Series will end in 6 games? (0.1984)
13. An airline knows that an individual making a reservation on a certain flight is only 95% likely to show up. Consequently, their policy is to sell 52 tickets for a flight that can only hold 50 passengers.
- (a) What is the probability that there will be a seat available for every passenger that shows up? (0.7405)
 - (b) What is the probability that the airline oversells the flight? (0.2595)\