STAT 213 L05

Solutions to Assignment #3

3.130 First, we find the following probabilities:

$$P(A \cap B_1) = P(A|B_1)P(B_1) = .4(.2) = .08$$
$$P(A \cap B_2) = P(A|B_2)P(B_2) = .25(.15) = .0375$$
$$P(A \cap B_3) = P(A|B_3)P(B_3) = .6(.65) = .5075$$
$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) = .08 + .0375 + .39 = .5075$$

a. $P(B_1 | A) = \dfrac{P(A \cap B_1)}{P(A)} = \dfrac{.08}{.5075} = .158$

b. $P(B_2 | A) = \dfrac{P(A \cap B_2)}{P(A)} = \dfrac{.0375}{.5075} = .074$

c. $P(B_3 | A) = \dfrac{P(A \cap B_3)}{P(A)} = \dfrac{.39}{.5075} = .768$

3.132 a. Converting the percentages to probabilities,

$P(275 - 300) = .52$, $P(305 - 325) = .39$, and $P(330 - 350) = .09$.

b. Using Bayes Theorem,

$$P(275-300 | CC) = \frac{P(275-300 \cap CC)}{P(CC)}$$

$$= \frac{P(CC | 275-300)P(275-300)}{P(CC | 275-300)P(275-300) + P(CC | 305-325)P(305-325) + P(CC | 330-350)P(330-350)}$$

$$= \frac{.775(.52)}{.775(.52) + .77(.39) + .86(.09)} = \frac{.403}{.403 + .3003 + .0774} = \frac{.403}{.7807} = .516$$

3.134 a.

$$P(E_1 \mid error) = \frac{P(E_1 \cap error)}{P(error)}$$

$$= \frac{P(error \mid E_1)P(E_1)}{P(error \mid E_1)P(E_1) + P(error \mid E_2)P(E_2) + P(error \mid E_3)P(E_3)}$$

$$= \frac{.01(.30)}{.01(.30) + .03(.20) + .02(.50)} = \frac{.003}{.003 + .006 + .01} = \frac{.003}{.019} = .158$$

b.

$$P(E_2 \mid error) = \frac{P(E_2 \cap error)}{P(error)}$$

$$= \frac{P(error \mid E_2)P(E_2)}{P(error \mid E_1)P(E_1) + P(error \mid E_2)P(E_2) + P(error \mid E_3)P(E_3)}$$

$$= \frac{.03(.20)}{.01(.30) + .03(.20) + .02(.50)} = \frac{.006}{.003 + .006 + .01} = \frac{.006}{.019} = .316$$

c.

$$P(E_3 \mid error) = \frac{P(E_3 \cap error)}{P(error)}$$

$$= \frac{P(error \mid E_3)P(E_3)}{P(error \mid E_1)P(E_1) + P(error \mid E_2)P(E_2) + P(error \mid E_3)P(E_3)}$$

$$= \frac{.02(.50)}{.01(.30) + .03(.20) + .02(.50)} = \frac{.01}{.003 + .006 + .01} = \frac{.01}{.019} = .526$$

d. If there was a serious error, the probability that the error was made by engineer 3 is .526. This probability is higher than for any of the other engineers. Thus engineer #3 is most likely responsible for the error.

4.30 a. $\mu = E(x) = \sum xp(x)$

$$= 10(.05) + 20(.20) + 30(.30) + 40(.25) + 50(.10) + 60(.10)$$
$$= .5 + 4 + 9 + 10 + 5 + 6 = 34.5$$

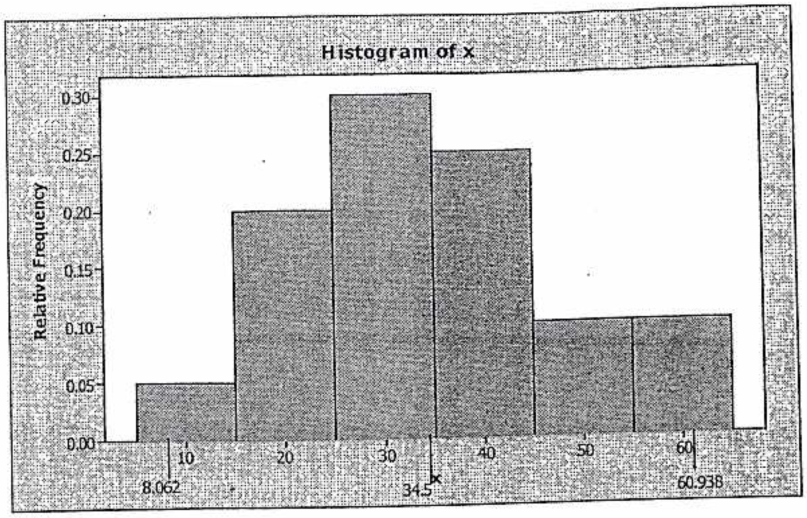
$$\sigma^2 = E(x-\mu)^2 = \sum (x-\mu)^2 p(x)$$
$$= (10 - 34.5)^2(.05) + (20 - 34.5)^2(.20) + (30 - 34.5)^2(.30) + (40 - 34.5)^2(.25) + (50 - 34.5)^2(.10) + (60 - 34.5)^2(.10)$$
$$= 30.0125 + 42.05 + 6.075 + 7.5625 + 24.025 + 65.025 = 174.75$$

$$\sigma = \sqrt{174.75} = 13.219$$

(continued)

4.30 (continued)

b.



c. $\mu \pm 2\sigma \Rightarrow 34.5 \pm 2(13.219) \Rightarrow 34.5 \pm 26.438 \Rightarrow (8.062, 60.938)$

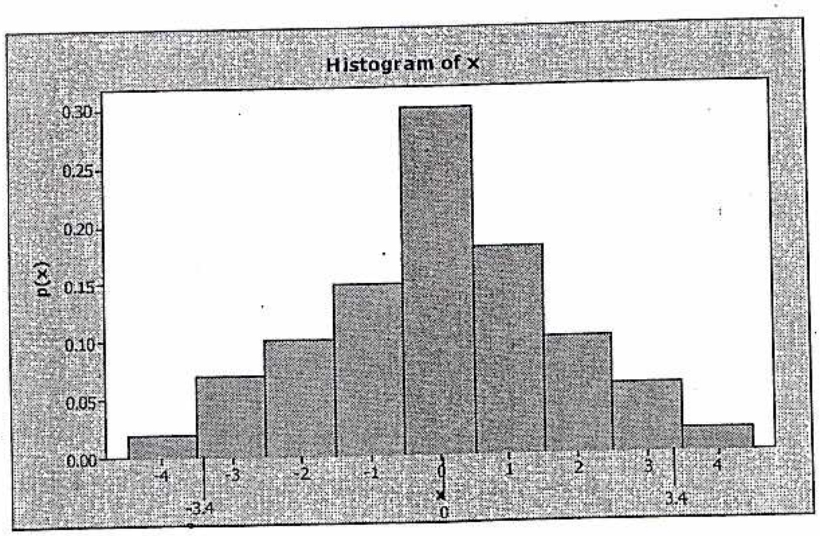
$$P(8.062 < x < 60.938) = p(10) + p(20) + p(30) + p(40) + p(50) + p(60)$$
$$= .05 + .20 + .30 + .25 + .10 + .10 = 1.00$$

4.32 a. $\mu = E(x) = \sum xp(x) = -4(.02) + (-3)(.07) + (-2)(.10) + (-1)(.15) + 0(.3)$
$$+ 1(.18) + 2(.10) + 3(.06) + 4(.02)$$
$$= -.08 - .21 - .2 - .15 + 0 + .18 + .2 + .18 + .08 = 0$$

$$\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2 p(x)$$
$$= (-4-0)^2(.02) + (-3-0)^2(.07) + (-2-0)^2(.10)$$
$$+ (-1-0)^2(.15) + (0-0)^2(.30) + (1-0)^2(.18)$$
$$+ (2-0)^2(.10) + (3-0)^2(.06) + (4-0)^2(.02)$$
$$= .32 + .63 + .4 + .15 + 0 + .18 + .4 + .54 + .32 = 2.94$$

$\sigma = \sqrt{2.94} = 1.715$

b.



$\mu \pm 2\sigma \Rightarrow 0 \pm 2(1.715) \Rightarrow 0 \pm 3.430 \Rightarrow (-3.430, 3.430)$

c. $P(-3.430 < x < 3.430) = p(-3) + p(-2) + p(-1) + p(0) + p(1) + p(2) + p(3)$
$$= .07 + .10 + .15 + .30 + .18 + .10 + .06 = .96$$

4.36 a. $E(x) = \sum xp(x) = 0(.09) + 1(.30) + 2(.37) + 3(.20) + 4(.04) = 0 + .30 + .74 + .60 + .16 = 1.8$

In a random samples of 4 homes, the average number of homes with high dust mite levels is 1.8.

b. $\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2\, p(x)$
$= (0-1.8)^2\,(.09) + (1-1.8)^2(.30) + (2-1.8)^2(.37) + (3-1.8)^2(.20) + (4-1.8)^2(.04)$

$= .2916 + .1920 + .0148 + .2880 + .1936 = .98$

$\sigma = \sqrt{.98} = .9899$

c. $\mu \pm 2\sigma \Rightarrow 1.8 \pm 2(.9899) \Rightarrow 1.8 \pm 1.9798 \Rightarrow (-.1798, 3.7798)$

$P(-.1798 < x < 3.7798) = p(0) + p(1) + p(2) + p(3) = .09 + .30 + .37 + .20 = .96$

Chebyshev's Theorem says that the interval $\mu \pm 2\sigma$ will contain at least .75 of the data. The Empirical Rule says that approximately .95 of the data will be contained in the interval. Both Chebyshev's Theorem and the Empirical Rule fit the distribution.

4.39 For a \$5 bet, you will either win \$5 or lose \$5 (–\$5). The probability distribution for the net winnings is:

x	$p(x)$
–5	20/38
5	18/38

$$\mu = E(x) = \sum xp(x) = -5\left(\frac{20}{38}\right) + 5\left(\frac{18}{38}\right) = -.263$$

Over a large number of trials, the average winning for a \$5 bet on red is -\$0.263.

4.40 Let x = winnings in the Florida lottery. The probability distribution for x is:

x	$p(x)$
–\$1	13,999,999/14,000,000
\$6,999,999	1/14,000,000

The expected net winnings would be:

$$\mu = E(x) = (-1)(13{,}999{,}999/14{,}000{,}000) + 6{,}999{,}999(1/14{,}000{,}000) = -\$.50$$

The average winnings of all those who play the lottery is –\$.50.

4.41

a. Since there are 20 possible outcomes that are all equally likely, the probability of any of the 20 numbers is 1/20. The probability distribution of x is:

$P(x = 5) = 1/20 = .05$; $P(x = 10) = 1/20 = .05$; etc.

x	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
$p(x)$	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05

b. $E(x) = \sum xp(x) = 5(.05) + 10(.05) + 15(.05) + 20(.05) + 25(.05) + 30(.05) + 35(.05)$
$+ 40(.05) + 45(.05) + 50(.05) + 55(.05) + 60(.05) + 65(.05) + 70(.05) + 75(.05)$
$+ 80(.05) + 85(.05) + 90(.05) + 95(.05) + 100(.05) = 52.5$

c. $\sigma^2 = E(x - \mu)^2 = \sum (x-\mu)^2 p(x) = (5 - 52.5)^2(.05) + (10 - 52.5)^2(.05)$
$+ (15 - 52.5)^2(.05) + (20 - 52.5)^2(.05) + (25 - 52.5)^2(.05) + (30 - 52.5)^2(.05)$
$+ (35 - 52.5)^2(.05) + (40 - 52.5)^2(.05) + (45 - 52.5)^2(.05) + (50 - 52.5)^2(.05)$
$+ (55 - 52.5)^2(.05) + (60 - 52.5)^2(.05) + (65 - 52.5)^2(.05) + (70 - 52.5)^2(.05)$
$+ (75 - 52.5)^2(.05) + (80 - 52.5)^2(.05) + (85 - 52.5)^2(.05) + (90 - 52.5)^2(.05)$
$+ (95 - 52.5)^2(.05) + (100 - 52.5)^2(.05)$
$= 831.25$

$\sigma = \sqrt{\sigma^2} = \sqrt{831.25} = 28.83$

Since the uniform distribution is not mound-shaped, we will use Chebyshev's theorem to describe the data. We know that at least 8/9 of the observations will fall with 3 standard deviations of the mean and at least 3/4 of the observations will fall within 2 standard deviations of the mean. For this problem,

$\mu \pm 2\sigma \Rightarrow 52.5 \pm 2(28.83) \Rightarrow 52.5 \pm 57.66 \Rightarrow (-5.16, 110.16)$. Thus, at least 3/4 of the data will fall between -5.16 and 110.16. For our problem, all of the observations will fall within 2 standard deviations of the mean. Thus, x is just as likely to fall within any interval of equal length..

d. If a player spins the wheel twice, the total number of outcomes will be 20(20) = 400. The sample space is:

5, 5	10, 5	15, 5	20, 5	25, 5...	100, 5
5,10	10,10	15,10	20,10	25,10...	100,10
5,15	10,15	15,15	20,15	25,15...	100,15
⋮	⋮	⋮	⋮	⋮	⋮
5,100	10,100	15,100	20,100	25,100...	100,100

(continued)

4.41 (continued)
d. (continued)

Each of these outcomes are equally likely, so each has a probability of 1/400 = .0025.

Now, let x equal the sum of the two numbers in each sample. There is one sample with a sum of 10, two samples with a sum of 15, three samples with a sum of 20, etc. If the sum of the two numbers exceeds 100, then x is zero. The probability distribution of x is:

x	$p(x)$
0	.5250
10	.0025
15	.0050
20	.0075
25	.0100
30	.0125
35	.0150
40	.0175
45	.0200
50	.0225
55	.0250
60	.0275
65	.0300
70	.0325
75	.0350
80	.0375
85	.0400
90	.0425
95	.0450
100	.0475

e. We assumed that the wheel is fair, or that all outcomes are equally likely.

f. $\mu = E(x) = \sum xp(x) = 0(.5250) + 10(.0025) + 15(.0050) + 20(.0075) + \ldots + 100(.0475)$
$= 33.25$

$\sigma^2 = E(x-\mu)^2 = \sum (x-\mu)^2 p(x) = (0-33.25)^2(.525) + (10-33.25)^2(.0025)$
$+ (15-33.25)^2(.0050) + (20-33.25)^2(.0075) + \ldots + (100-33.25)^2(.0475)$
$= 1471.3125$

$\sigma = \sqrt{\sigma^2} = \sqrt{1471.3125} = 38.3577$

g. $P(x=0) = .525$

(continued)

4.41 (continued)

h. Given that the player obtains a 20 on the first spin, the possible values for x (sum of the two spins) are 0 (player spins 85, 90, 95, or 100 on the second spin), 25, 30, ..., 100. In order to get an x of 25, the player would spin a 5 on the second spin. Similarly, the player would have to spin a 10 on the second spin order to get an x of 30, etc. Since all of the outcomes are equally likely on the second spin, the distribution of x is:

x	$p(x)$
0	.20
25	.05
30	.05
35	.05
40	.05
45	.05
50	.05
55	.05
60	.05
65	.05
70	.05
75	.05
80	.05
85	.05
90	.05
95	.05
100	.05

i. The probability that the players total score will exceed one dollar is the probability that x is zero. $P(x = 0) = .20$

j. Given that the player obtains a 65 on the first spin, the possible values for x (sum of the two spins) are 0 (player spins 40, 45, 50, up to 100 on second spin), 70, 75, 80,..., 100. In order to get an x of 70, the player would spin a 5 on the second spin. Similarly, the player would have to spin a 10 on the second spin in order to get an x of 75, etc. Since all of the outcomes are equally likely on the second spin, the distribution of x is:

x	$p(x)$
0	.65
70	.05
75	.05
80	.05
85	.05
90	.05
95	.05
100	.05

The probability that the players total score will exceed one dollar is the probability that x is zero. $P(x = 0) = .65$.

4.50 a. $p(0) = \binom{3}{0}(.3)^0(.7)^{3-0} = \frac{3!}{0!3!}(.3)^0(.7)^3 = \frac{3\cdot 2\cdot 1}{1\cdot 3\cdot 2\cdot 1}(1)(.7)^3 = .343$

$p(1) = \binom{3}{1}(.3)^1(.7)^{3-1} = \frac{3!}{1!2!}(.3)^1(.7)^2 = .441$

$p(2) = \binom{3}{2}(.3)^2(.7)^{3-2} = \frac{5!}{2!1!}(.3)^2(.7)^1 = .189$

$p(3) = \binom{3}{3}(.3)^3(.7)^{3-3} = \frac{5!}{3!0!}(.3)^3(.7)^0 = .027$

b.

x	$p(x)$
0	.343
1	.441
2	.189
3	.027

4.52 a. $P(x = 2) = P(x \le 2) - P(x \le 1) = .167 - .046 = .121$ (from Table II, Appendix A)

b. $P(x \le 5) = .034$

c. $P(x > 1) = 1 - P(x \le 1) = 1 - .919 = .081$

4.54 a. The simple events listed below are all equally likely, implying a probability of 1/32 for each. The list is in a regular pattern such that the first simple event would yield $x = 0$, the next five yield $x = 1$, the next ten yield $x = 2$, the next ten also yield $x = 3$, the next five yield $x = 4$, and the final one yields $x = 5$. The resulting probability distribution is given below the simple events.

$$\begin{bmatrix} \text{FFFFF, FFFFS, FFFSF, FFSFF, FSFFF, SFFFF, FFFSS, FFSFS} \\ \text{FSFFS, SFFFS, FFSSF, FSFSF, SFFSF, FSSFF, SFSFF, SSFFF} \\ \text{FFSSS, FSFSS, SFFSS, FSSFS, SFSFS, SSFFS, FSSSF, SFSSF} \\ \text{SSFSF, SSSFF, FSSSS, SFSSS, SSFSS, SSSFS, SSSSF, SSSSS} \end{bmatrix}$$

x	0	1	2	3	4	5
$p(x)$	1/32	5/32	10/32	10/32	5/32	1/32

(continued)

4.54 (continued)

b. $$P(x=0) = \frac{5!}{0!5!}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(1)(5\cdot4\cdot3\cdot2\cdot1)}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5$$
$$= 1\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 = \frac{1}{32} = .03125$$

$$P(x=1) = \frac{5!}{1!4!}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(1)(4\cdot3\cdot2\cdot1)}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4$$
$$= 5\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 = \frac{5}{32} = .15625$$

$$P(x=2) = \frac{5!}{2!3!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(2\cdot1)(3\cdot2\cdot1)}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3$$
$$= 10\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = \frac{10}{32} = .3125$$

$$P(x=3) = \frac{5!}{3!2!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(2\cdot1)}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$$
$$= 10\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{10}{32} = .3125$$

$$P(x=4) = \frac{5!}{4!1!}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(4\cdot3\cdot2\cdot1)(1)}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1$$
$$= 5\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5}{32} = .15625$$

$$P(x=5) = \frac{5!}{5!0!}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(5\cdot4\cdot3\cdot2\cdot1)(1)}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$$
$$= 1\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 = \frac{1}{32} = .15625$$

c. From Table II, $n = 5$, $p = .5$:

$P(x=0) = .031$
$P(x=1) = .188 - .031 = .157$
$P(x=2) = .500 - .188 = .312$
$P(x=3) = .812 - .500 = .312$
$P(x=4) = .969 - .812 = .157$
$P(x=5) = 1 - .969 = .031$

4.56 a. To see if x is a binomial random variable, we must check the 5 characteristics:

1. There are 200 identical trials.
2. There are 2 possible outcomes – a young adult owns a mobile phone with internet access (S) or not (F).
3. The probability of S is p = .20 for each trial.
4. The trials (students) are independent.
5. Let x = number of young adults who own a mobile phone with internet access in 200 trials.

Since all the characteristics are met, x is a binomial random variable.

b. The value of p is p = .20.

c. $\mu = E(x) = np = 200(.2) = 40$

4.60 a. From the problem, x is a binomial random variable with $n = 3$ and $p = .6$.

$$P(x=0) = \binom{3}{0}.6^0\,.4^{3-0} = \frac{3!}{0!3!}.6^0.4^3 = .064.$$

b. $P(x \geq 1) = 1 - P(x=0) = 1 - .064 = .936.$

c. $\mu = E(x) = np = 3\,(.6) = 1.8$

$\sigma = \sqrt{npq} = \sqrt{3(.6)(.4)} = .8485$

In samples of 3 parents, on the average, 1.8 condone spanking.

4.62 a. Let x = number of democratic regimes that allow a free press in 50 trials. For this problem, $p = .8$.

$\mu = E(x) = np = 50(.8) = 40$

We would expect 40 democratic regimes out of the 50 to have a free press.
$\sigma = \sqrt{np(1-p)} = \sqrt{50(.8)(.2)} = 2.828$

We would expect most observations to fall within 2 standard deviations of the mean:

$\mu \pm 2\sigma \Rightarrow 40 \pm 2(2.828) \Rightarrow 40 \pm 5.656 \Rightarrow (34.344, 45.656)$

We would expect to see anywhere between 35 and 45 democratic regimes to have a free press out of a sample of 50.

(continued)

4.62 (continued)

b. Let x = number of non-democratic regimes that allow a free press in 50 trials. For this problem, $p = .1$.

$\mu = E(x) = np = 50(.1) = 5$

We would expect 5 non-democratic regimes out of the 50 to have a free press.

$\sigma = \sqrt{np(1-p)} = \sqrt{50(.1)(.9)} = 2.121$

We would expect most observations to fall within 2 standard deviations of the mean:

$\mu \pm 2\sigma \Rightarrow 5 \pm 2(2.121) \Rightarrow 5 \pm 4.243 \Rightarrow (0.757, 9.243)$

We would expect to see anywhere between 1 to 9 non-democratic regimes to have a free press out of a sample of 50.

4.64 Let x = number of slaughtered chickens in 5 that passes inspection with fecal contamination. Then x is a binomial random variable with $n = 5$ and $p = .01$ (from Exercise 3.15.)

$P(x \geq 1) = 1 - P(x = 0) = 1 - .951 = .049$ (From Table II, Appendix A).