STAT 213 L05

Solutions to Assignment # 7

8.24 a. H_0: $\mu = .36$
H_a: $\mu < .36$

The test statistic is $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{.323 - .36}{\sqrt{.034} / \sqrt{64}} = -1.61$

The rejection region requires $\alpha = .10$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = -1.61 < -1.28$), H_0 is rejected. There is sufficient evidence to indicate the mean is less than .36 at $\alpha = .10$.

b. H_0: $\mu = .36$
H_a: $\mu \neq .36$

The test statistic is $z = -1.61$ (see part a).

The rejection region requires $\alpha/2 = .10/2 = .05$ in the each tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.61 \nless -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from .36 at $\alpha = .10$.

8.26 a. The rejection region requires $\alpha = .01$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.01} = 2.33$. The rejection region is $z < -2.33$.

b. The test statistic is $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{19.3 - 20}{\frac{11.9}{\sqrt{46}}} = -.40$

c. Since the observed value of the test statistic does not fall in the rejection region ($z = -.40 \nless -2.33$), H_0 is not rejected. There is insufficient evidence to indicate the mean number of latex gloves used per week by hospital employees diagnosed with a latex allergy from exposure to the powder on latex gloves is less than 20 at $\alpha = .01$.

8.34 a. To determine if the true mean cyanide level in soil in The Netherlands exceeds 100mg/kg, we test:

H_0: $\mu = 100$
H_a: $\mu > 100$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{84 - 100}{80/\sqrt{72}} = -1.70$

The rejection region requires $\alpha = .10$ in the upper tail of the z distribution. From Table IV, Appendix A, $z_{.10} = 1.28$. The rejection region is $z > 1.28$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.70 \not> 1.28$), H_0 is not rejected. There is insufficient evidence to indicate that the true mean cyanide level in soil in The Netherlands exceeds 100 mg/kg $\alpha = .10$.

b. The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.70 \not> 1.645$), H_0 is not rejected. There is insufficient evidence to indicate that the true mean cyanide level in soil in The Netherlands exceeds 100 mg/kg $\alpha = .05$.

The rejection region requires $\alpha = .01$ in the upper tail of the z distribution. From Table IV, Appendix A, $z_{.01} = 2.33$. The rejection region is $z > 2.33$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.70 \Uparrow 2.33$), H_0 is not rejected. There is insufficient evidence to indicate that the true mean cyanide level in soil in The Netherlands exceeds 100 mg/kg $\alpha = .01$.

For this problem, since the test statistic falls on the opposite side of the mean than the rejection region, we will not reject H_0 for any reasonable value of α.

If the value of α changes, the conclusion could also change since the rejection region changes.

8.58 For this sample,

$$\bar{x} = \frac{\sum x}{n} = \frac{11}{6} = 1.8333 \qquad s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{41 - \frac{11^2}{6}}{6-1} = 4.1667$$

$$s = \sqrt{s^2} = 2.0412$$

a. H_0: $\mu = 3$
H_a: $\mu < 3$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{1.8333 - 3}{2.0412/\sqrt{6}} = -1.40$

(continued)

8.58 (continued)

The rejection region requires $\alpha = .05$ in the lower tail of the t distribution with $df = n - 1 = 6 - 1 = 5$. From Table VI, Appendix A, $t_{.05} = 2.015$. The rejection region is $t < -2.015$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.40 \nless -2.015$), H_0 is not rejected. There is insufficient evidence to indicate μ is less than 3 at $\alpha = .05$.

b. H_0: $\mu = 3$
H_a: $\mu \neq 3$

Test statistic: $t = -1.40$ (Refer to part **a**.)

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the t distribution with $df = n - 1 = 6 - 1 = 5$. From Table VI, Appendix A, $t_{.025} = 2.571$. The rejection region is $t < -2.571$ or $t > 2.571$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.40 \nless -2.571$), H_0 is not rejected. There is insufficient evidence to indicate μ differs from 3 at $\alpha = .05$.

c. For part **a**: p-value $= P(t \leq -1.40)$

From Table VI, with $df = 5$, $P(t \leq -1.40) > .10$

For part **b**: p-value $= P(t \leq -1.40) + P(t \geq 1.40)$

From Table VI, with $df = 5$, p-value $= 2P(t \geq 1.40) > 2(.10) = .20$

8.61 a. To determine if the mean breaking strength of the new bonding adhesive is less than 5.70 Mpa, we test:

H_0: $\mu = 5.70$
H_a: $\mu < 5.70$

b. The rejection region requires $\alpha = .01$ in the lower tail of the t distribution with $df = n - 1 = 10 - 1 = 9$. From Table VI, Appendix A, $t_{.01} = 2.821$. The rejection region is $t < -2.821$.

c. The test statistic is $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{5.07 - 5.70}{\frac{.46}{\sqrt{10}}} = -4.33$.

d. Since the observed value of the test statistic falls in the rejection region ($t = -4.33 < -2.821$), H_o is rejected. There is sufficient evidence to indicate that the mean breaking strength of the new bonding adhesive is less than 5.70 Mpa at $\alpha = .01$.

e. The conditions required for this test is a random sample from the target population and the population from which the sample is selected is approximately normal.

8.62 a. To determine if the average number of books read by all students who participate in the extensive reading program exceeds 25, we test:

H_0: $\mu = 25$
H_a: $\mu > 25$

b. The rejection region requires $\alpha = .05$ in the upper tail of the t distribution with df $= n - 1 = 14 - 1 = 13$. From Table VI, Appendix A, $t_{.05} = 1.771$. The rejection region is $t > 1.771$.

c. The test statistic is $t = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \dfrac{31.64 - 25}{\frac{10.485}{\sqrt{14}}} = 2.37$

d. Since the observed value of the test statistic falls in the rejection region ($t = 2.37 > 1.771$), H_0 is rejected. There is sufficient evidence to indicate that the average number of books read by all students who participate in the extensive reading program exceeds 25 at $\alpha = .05$.

e. The conditions required for this test is a random sample from the target population and the population from which the sample is selected is approximately normal.

8.64 To determine if the true mean pouring temperature differs from the target setting, we test:

H_0: $\mu = 2{,}550$
H_a: $\mu \neq 2{,}550$

The test statistic is $t = 1.210$.

The p-value of the test is p = .2573. Since the p-value is greater than $\alpha = .01$, H_0 is not rejected. There is insufficient evidence to indicate the true mean pouring temperature differs from the target setting at $\alpha = .01$.

8.66 a. To determine if the mean repellency percentage of the new mosquito repellent is less than 95, we test:

H_0: $\mu = 95$
H_a: $\mu < 95$

The test statistic is $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}} = \dfrac{83 - 95}{15/\sqrt{5}} = -1.79$

The rejection region requires $\alpha = .10$ in the lower tail of the t distribution. From Table VI, Appendix A, with df $= n - 1 = 5 - 1 = 4$, $t_{.10} = 1.533$. The rejection region is $t < -1.533$.

Since the observed value of the test statistic falls in the rejection region ($t = -1.79 < -1.533$), H_0 is rejected. There is sufficient evidence to indicate that the true mean repellency percentage of the new mosquito repellent is less than 95 at $\alpha = .10$.

b. We must assume that the population of percent repellencies is normally distributed.

8.74 b. First, check to see if n is large enough.

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .75 \pm 3\sqrt{\frac{(.75)(.25)}{100}} \Rightarrow .75 \pm .13 \Rightarrow (.62, .88)$$

Since the interval lies within the interval (0, 1), the normal approximation will be adequate.

H_0: $p = .75$
H_a: $p < .75$

The test statistic is $z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.69 - .75}{\sqrt{\frac{.75(.25)}{100}}} = -1.39$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($-1.39 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is less than .75 at $\alpha = .05$.

c. p-value $= P(z \le -1.39) = .5 - .4177 = .0823$

8.78 a. The point estimate for p is $\hat{p} = \frac{x}{n} = \frac{64}{106} = .604$.

b. H_0: $p = .70$
H_a: $p \neq .70$

c. The test statistic is $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.604 - .70}{\sqrt{\frac{.7(.3)}{106}}} = -2.16$

d. The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.005} = 2.58$. The rejection region is $z < -2.58$ or $z > 2.58$.

e. First, check to see if the normal approximation will be adequate:

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{pq}{n}} \approx p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .7 \pm 3\sqrt{\frac{.7(.3)}{106}} \Rightarrow .7 \pm .134 \Rightarrow (.566,\ \ .834)$$

Since the interval is completely in the interval (0, 1), the normal approximation will be adequate.

Since the observed value of the test statistic does not fall in the rejection region ($z = -2.16 \not< -2.58$), H_0 is not rejected. There is insufficient evidence to indicate the proportion of consumers who believe "Made in the USA" means 100% of labor and materials are from the United States is different from .70 at $\alpha = .01$.

8.80 a. The population parameter of interest is the proportion of all television viewers who agree with the statement "Overall, I find the quality of news on cable networks (such as CN, FOXNews, CNBC, and MSNBC) to be better than news on the ABC, CBS, and NBC networks."

b. The point estimate for p is $\hat{p} = \frac{x}{n} = \frac{248}{500} = .496$.

c. To determine if the true percentage of TV-viewers who find cable news to be better quality than network news differs from 50%, we test:

H_0: $p = .50$
H_a: $p \neq .50$

d. First, check to see if the normal approximation will be adequate:

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{pq}{n}} \approx p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .50 \pm 3\sqrt{\frac{.50(.50)}{500}} \Rightarrow .50 \pm .067 \Rightarrow (.433,\ \ .567)$$

Since the interval is completely in the interval (0, 1), the normal approximation will be adequate.

The test statistic is $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.496 - .50}{\sqrt{\frac{.5(.5)}{500}}} = -0.18$

The rejection region requires $\alpha/2 = .10/2 = .05$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -.18 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the true percentage of TV-viewers who find cable news to be better quality than network news differs from 50% at $\alpha = .10$.

e. The required conditions are that a random sample is selected and the sample size is large enough. Both of these conditions appear to be met. (The sample size was checked in part d.)

8.86 **For the Top of the Core:**

First, check to see if the normal approximation is adequate:

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .5 \pm 3\sqrt{\frac{(.5)(.5)}{84}} \Rightarrow .5 \pm\ .164 \Rightarrow (.336, .664)$$

Since the interval falls completely in the interval (0,1), the normal distribution will be adequate.

$$\hat{p} = \frac{x}{n} = \frac{64}{84} = .762$$

To determine if the coat index exceeds .5, we test:

H_0: $p = .5$
H_a: $p > .5$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.762 - .5}{\sqrt{\dfrac{.5(.5)}{84}}} = 4.80$

The rejection region requires $\alpha = .05$ in the upper tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z > 1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = 4.80 > 1.645$), H_0 is rejected. There is sufficient evidence to indicate that the coat index exceeds .5 at $\alpha = .05$.

For the Middle of the Core:

First, check to see if the normal approximation is adequate:

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .5 \pm 3\sqrt{\frac{(.5)(.5)}{73}} \Rightarrow .5 \pm .176 \Rightarrow (.324, .676)$$

Since the interval falls completely in the interval (0,1), the normal distribution will be adequate.

$$\hat{p} = \frac{x}{n} = \frac{35}{73} = .479$$

To determine if the coat index differs from .5, we test:

H_0: $p = .5$
H_a: $p \neq .5$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.479 - .5}{\sqrt{\dfrac{.5(.5)}{73}}} = -.36$

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

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8,86 (continued)

Since the observed value of the test statistic does not fall in the rejection region ($z = -.36 \not< -1.96$), H_0 is not rejected. There is insufficient evidence to indicate that the coat index differs from .5 at $\alpha = .05$.

For the Bottom of the Core:

First, check to see if the normal approximation is adequate:

$$p_0 \pm 3\sigma_{\hat{p}} \Rightarrow p_0 \pm 3\sqrt{\frac{p_0 q_0}{n}} \Rightarrow .5 \pm 3\sqrt{\frac{(.5)(.5)}{81}} \Rightarrow .5 \pm .167 \Rightarrow (.333, .667)$$

Since the interval falls completely in the interval (0,1), the normal distribution will be adequate.

$$\hat{p} = \frac{x}{n} = \frac{29}{81} = .358$$

To determine if the coat index is less than .5, we test:

H_0: $p = .5$
H_a: $p < .5$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{.358 - .5}{\sqrt{\dfrac{.5(.5)}{81}}} = -2.56$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = -2.56 < -1.645$), H_0 is rejected. There is sufficient evidence to indicate that the coat index is less than .5 at $\alpha = .05$.