# STAT 213 L05

## Solutions to Assignment #2

11.14 a.

| $x_i$ | $y_i$ | $x_i^2$ | $x_iy_i$ |
|---|---|---|---|
| 7 | 2 | $7^2 = 49$ | $7(2) = 14$ |
| 4 | 4 | $4^2 = 16$ | $4(4) = 16$ |
| 6 | 2 | $6^2 = 36$ | $6(2) = 12$ |
| 2 | 5 | $2^2 = 4$ | $2(5) = 10$ |
| 1 | 7 | $1^2 = 1$ | $1(7) = 7$ |
| 1 | 6 | $1^2 = 1$ | $1(6) = 6$ |
| 3 | 5 | $3^2 = 9$ | $3(5) = 15$ |

Totals: $\sum x_i = 7 + 4 + 6 + 2 + 1 + 1 + 3 = 24$

$\sum y_i = 2 + 4 + 2 + 5 + 7 + 6 + 5 = 31$

$\sum x_i^2 = 49 + 16 + 36 + 4 + 1 + 1 + 9 = 116$

$\sum x_i y_i = 14 + 16 + 12 + 10 + 7 + 6 + 15 = 80$

b. $$SS_{xy} = \sum x_i y_i = \frac{\left(\sum x_i\right)\left(\sum y_i\right)}{n} = 80 - \frac{(24)(31)}{7} = 80 - 106.2857143 = -26.2857143$$

c. $$SS_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{7} = 116 - \frac{(24)^2}{7} = 116 - 82.28571429 = 33.71428571$$

d. $$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{-26.2857143}{33.71428571} = -.779661017 \approx -.7797$$

e. $$\bar{x} = \frac{\sum x_i}{n} = \frac{24}{7} = 3.428571429 \qquad \bar{y} = \frac{\sum y_i}{n} = \frac{31}{7} = 4.428571429$$

f. $$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 4.428571429 - (-.779661017)(3.428571429)$$
$$= 4.428571429 - (-2.673123487) = 7.101694916 \approx 7.102$$

g. The least squares line is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 7.102 - .7797x$.

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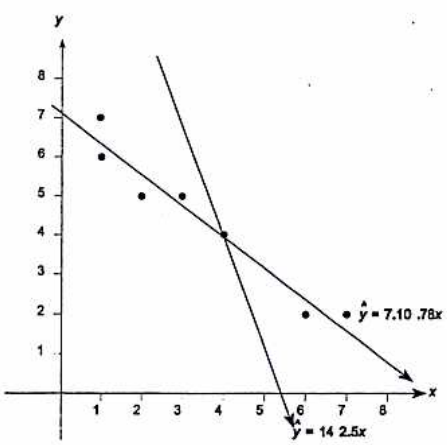
11.15 From Exercise 11.14, $\hat{\beta}_0 = 7.10$ and $\hat{\beta}_1 = -.78$.

The fitted line is $= 7.10 - .78x$. To obtain values for , we substitute values of $x$ into the equation and solve for.

a.

| $x$ | $y$ | $\hat{y} = 7.10 - .78x$ | $(y - \hat{y})$ | $(y - \hat{y})^2$ |
|---|---|---|---|---|
| 7 | 2 | 1.64 | .36 | .1296 |
| 4 | 4 | 3.98 | .02 | .0004 |
| 6 | 2 | 2.42 | −.42 | .1764 |
| 2 | 5 | 5.54 | −.54 | .2916 |
| 1 | 7 | 6.32 | .68 | .4624 |
| 1 | 6 | 6.32 | −.32 | .1024 |
| 3 | 5 | 4.76 | .24 | .0576 |
| | | | $\sum(y-\hat{y}) = 0.02$ | SSE $= \sum(y-\hat{y})^2 = 1.2204$ |

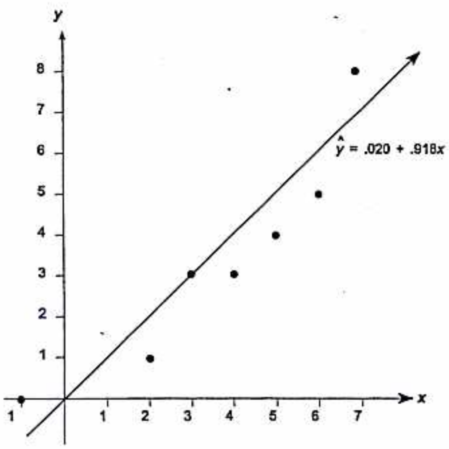
b.




c.

| $x$ | $y$ | $\hat{y} = 14 - 2.5x$ | $(y - \hat{y})$ | $(y - \hat{y})^2$ |
|---|---|---|---|---|
| 7 | 2 | −3.5 | 5.5 | 30.25 |
| 4 | 4 | 4 | 0 | 0 |
| 6 | 2 | −1 | 3 | 9 |
| 2 | 5 | 9 | −4 | 16 |
| 1 | 7 | 11.5 | −4.5 | 20.25 |
| 1 | 6 | 11.5 | −5.5 | 30.25 |
| 3 | 5 | 6.5 | −1.5 | 2.25 |
| | | | = −7 | SSE = 108.00 |

11.17 a.




b. As $x$ increases, $y$ tends to increase. Thus, there appears to be a positive, linear relationship between $y$ and $x$.

c. $\hat{\beta}_1 = \dfrac{SS_{xy}}{SS_{xx}} = \dfrac{39.8571}{43.4286} = .9177616 \approx .918$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 3.4286 - .9177616(3.7143) = .0197581 \approx .020$

d. The line appears to fit the data quite well.

e. $\hat{\beta}_0 = .020$ The estimated mean value of $y$ when $x = 0$ is .020.

$\hat{\beta}_1 = .918$ The estimated change in the mean value of $y$ for each unit change in $x$ is .918.

These interpretations are valid only for values of $x$ in the range from −1 to 7.

11.66 $$r = \frac{SS_{xy}}{\sqrt{SS_{xx}\,SS_{yy}}}$$

| $y_i$ | $y_i^2$ |
|---|---|
| 2 | 4 |
| 4 | 16 |
| 2 | 4 |
| 5 | 25 |
| 7 | 49 |
| 6 | 36 |
| 5 | 25 |
| Σ: 31 | 159 |

$$SS_{yy} = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n} = 159 - \frac{(31)^2}{7}$$

$$= 21.714$$

Using $SS_{xy}$ and $SS_{xx}$ from 11.14 yields

$$r = \frac{-26.286}{\sqrt{(33.714)(21.714)}} = -0.9715$$

so $r^2 = (0.9715)^2 = 0.9438$

11.68 a. Some preliminary calculations are:

$$\sum x = 0 \qquad \sum x^2 = 10 \qquad \sum xy = 20$$
$$\sum y = 12 \qquad \sum y^2 = 70$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 20 - \frac{0(12)}{5} = 20$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 10 - \frac{0^2}{5} = 10$$

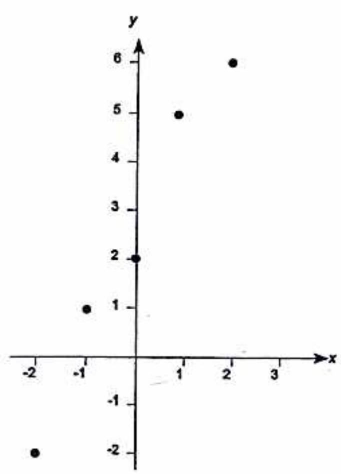
$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 70 - \frac{12^2}{5} = 41.2$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{20}{\sqrt{10(41.2)}} = .9853$$

$$r^2 = .9853^2 = .9709$$

Since $r = .9853$, there is a very strong positive linear relationship between $x$ and $y$.

Since $r^2 = .9709$, 97.09% of the total sample variability around  is explained by the linear relationship between $x$ and $y$.




b. Some preliminary calculations are:

$$\sum x = 0 \qquad \sum x^2 = 10 \qquad \sum xy = -15$$
$$\sum y = 16 \qquad \sum y^2 = 74$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = -15 - \frac{0(16)}{5} = -15$$

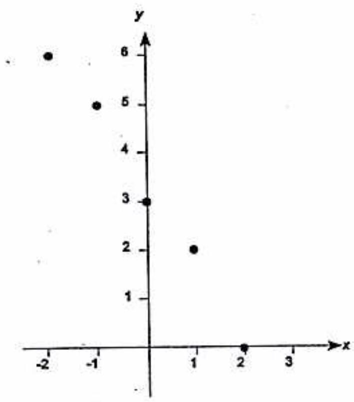
$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 10 - \frac{0^2}{5} = 10$$

11.68 (continued)

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 74 - \frac{16^2}{5} = 22.8$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{-15}{\sqrt{10(22.8)}} = -.9934$$

$$r^2 = (-.9934)^2 = .9868$$




Since $r = -.9934$, there is a very strong negative linear relationship between $x$ and $y$.

Since $r^2 = .9868$, 98.68% of the total sample variability around is explained by the linear relationship between $x$ and $y$.

c. Some preliminary calculations are:

$$\sum x = 18 \qquad \sum x^2 = 52 \qquad \sum xy = 36$$

$$\sum y = 14 \qquad \sum y^2 = 32$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 36 - \frac{18(14)}{7} = 0$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 52 - \frac{18^2}{7} = 5.71428571$$

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 32 - \frac{14^2}{7} = 4$$

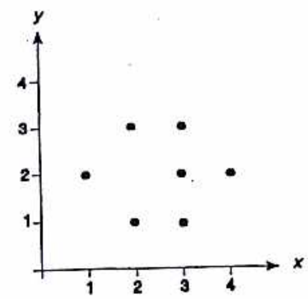
$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{0}{\sqrt{5.71428571(4)}} = 0$$

$$r^2 = 0^2 = 0$$

Since $r = 0$, this implies that $x$ and $y$ are not related.

Since $r^2 = 0$, 0% of the total sample variability around is explained by the linear relationship between $x$ and $y$.

11.68 (continued)




d. Some preliminary calculations are:

$$\sum x = 15 \qquad \sum x^2 = 71 \qquad \sum xy = 12$$
$$\sum y = 4 \qquad \sum y^2 = 6$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 12 - \frac{15(4)}{5} = 0$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 71 - \frac{15^2}{5} = 26$$

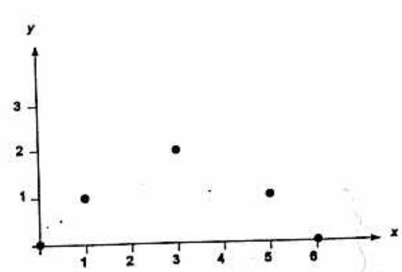
$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 6 - \frac{4^2}{5} = 2.8$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{0}{\sqrt{26(2.8)}} = 0$$

$$r^2 = 0^2 = 0$$

Since $r = 0$, this implies that $x$ and $y$ are not related.

Since $r^2 = 0$, 0% of the total sample variability around  is explained by the linear relationship between $x$ and $y$.



3.9 a. Since the probabilities must sum to 1,

$$P(E_3) = 1 - P(E_1) - P(E_2) - P(E_4) - P(E_5) = 1 - .1 - .2 - .1 - .1 = .5$$

b. $P(E_3) = 1 - P(E_3) - P(E_2) - P(E_4) - P(E_5)$

$$\Rightarrow 2P(E_3) = 1 - .1 - .2 - .1 \Rightarrow 2P(E_3) = .6 \Rightarrow P(E_3) = .3$$

c. $P(E_3) = 1 - P(E_1) - P(E_2) - P(E_4) - P(E_5) = 1 - .1 - .1 - .1 - .1 = .6$

3.10 a. If the simple events are equally likely, then

$$P(1) = P(2) = P(3) = \cdots = P(10) = \frac{1}{10}$$

Therefore,

$$P(A) = P(4) + P(5) + P(6) = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10} = .3$$

$$P(B) = P(6) + P(7) = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = .2$$

b. $P(A) = P(4) + P(5) + P(6) = \frac{1}{20} + \frac{1}{20} + \frac{3}{20} = \frac{5}{20} = .25$

$$P(B) = P(6) + P(7) = \frac{3}{10} + \frac{3}{10} = \frac{6}{10} = .3$$

3.14 a. The sample space for this experiment would consist of pairs of digits, indicating the result on each of the two dice.

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

b. Each of the above sample points are equally likely, and each therefore has a probability of 1/36.

c. The probability of each event below can be obtained by counting the number of sample points which belong to the event and multiplying this amount by 1/36. This results in the following:

$$P(A) = \frac{1}{36} \qquad P(B) = \frac{18}{36} \qquad P(C) = \frac{6}{36}$$

$$P(D) = \frac{11}{36} \qquad P(E) = \frac{6}{36}$$

3.15 a. If we denote the marbles as $B_1$, $B_2$, $R_1$, $R_2$, $R_3$, then the ten equally likely sample points in the sample space would be:

$$S: \begin{bmatrix} (B_1,B_2),(B_1,R_1),(B_1,R_2),(B_1,R_3),(B_2,R_1) \\ (B_2,R_2),(B_2,R_3),(R_1,R_2),(R_1,R_3),(R_2,R_3) \end{bmatrix}$$

Notice that order is ignored, as the only concern is whether or not a marble is selected.

b. Each of these ten would be equally likely, implying that each occurs with a probability 1/10.

c. $P(A) = \frac{1}{10}$ $\quad P(B) = 6\left(\frac{1}{10}\right) = \frac{6}{10} = \frac{3}{5}$ $\quad P(C) = 3\left(\frac{1}{10}\right) = \frac{3}{10}$

3.41 a.

$A$: {HHH, HHT, HTH, THH, TTH, THT, HTT}
$B$: {HHH, TTH, THT, HTT}
$A \cup B$: {HHH, HHT, HTH, THH, TTH, THT, HTT}
$A^c$: {TTT}
$A \cap B$: {HHH, TTH, THT, HTT}

b. If the coin is fair, then each of the 8 possible outcomes are equally likely, with probability 1/8.

$P(A) = \frac{7}{8}$ $\quad P(B) = \frac{4}{8} = \frac{1}{2}$ $\quad P(A \cup B) = \frac{7}{8}$

$P(A^c) = \frac{1}{8}$ $\quad P(A \cap B) = \frac{4}{8} = \frac{1}{2}$

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{7}{8} + \frac{1}{2} - \frac{1}{2} = \frac{7}{8}$

d. No. $P(A \cap B) = \frac{1}{2}$ which is not 0.

3.45 a. $P(A) = .50 + .10 + .05 = .65$

b. $P(B) = .10 + .07 + .50 + .05 = .72$

c. $P(C) = .25$

d. $P(D) = .05 + .03 = .08$.

e. $P(A^c) = .25 + .07 + .03 = .35$ (Note: $P(A^c) = 1 - P(A) = 1 - .65 = .35$)

f. $P(A \cup B) = P(B) = .10 + .07 + .50 + .05 = .72$

g. $P(A \cap B) = P(A) = .50 + .10 + .05 = .65$

h. Two events are mutually exclusive if they have no sample points in common or if the probability of their intersection is 0.

$P(A \cap B) = .50 + .10 + .05 = .65$. Since this is not 0, $A$ and $B$ are not mutually exclusive.

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3.45 (continued)

$P(A \cap C) = 0$. Since this is 0, $A$ and $C$ are mutually exclusive.

$P(A \cap D) = .05$. Since this is not 0, $A$ and $D$ are not mutually exclusive.

$P(B \cap C) = 0$. Since this is 0, $B$ and $C$ are mutually exclusive.

$P(B \cap D) = .05$. Since this is not 0, $B$ and $D$ are not mutually exclusive.

$P(C \cap D) = 0$. Since this is 0, $C$ and $D$ are mutually exclusive.

3.46 Define the following events:

$E_1$: {3 heads}
$E_2$: {2 heads}
$E_3$: {1 heads}
$E_4$: {0 heads}

a. $A = E_1 \cup E_2 \cup E_3$

$$P(A) = P(E_1) + P(E_2) + P(E_3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$$

b. $A = E_4^c$ $\qquad P(A) = 1 - P(E_4) = 1 - \frac{1}{8} = \frac{7}{8}$

3.49 a. The event $A \cap B$ is the event the outcome is black and odd. The event is $A \cap B$: {11, 13, 15, 17, 29, 31, 33, 35}

b. The event $A \cup B$ is the event the outcome is black or odd or both. The event $A \cup B$ is {2, 4, 6, 8, 10, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 31, 33, 35, 1, 3, 5, 7, 9, 19, 21, 23, 25, 27}

c. Assuming all events are equally likely, each has a probability of 1/38.

$$P(A) = 18\left(\frac{1}{38}\right) = \frac{18}{38} = \frac{9}{19}$$

$$P(B) = 18\left(\frac{1}{38}\right) = \frac{18}{38} = \frac{9}{19}$$

$$P(A \cap B) = 8\left(\frac{1}{38}\right) = \frac{8}{38} = \frac{4}{19}$$

$$P(A \cup B) = 28\left(\frac{1}{38}\right) = \frac{28}{38} = \frac{14}{19}$$

$$P(C) = 18\left(\frac{1}{38}\right) = \frac{18}{38} = \frac{9}{19}$$

(continued)

3.49 (continued)

d. The event $A \cap B \cap C$ is the event the outcome is odd and black and low. The event $A \cap B \cap C$ is {11, 13, 15, 17}.

e. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{9}{19} + \frac{9}{19} - \frac{4}{19} = \frac{14}{19}$

f. $P(A \cap B \cap C) = 4\left(\frac{1}{38}\right) = \frac{4}{38} = \frac{2}{19}$

g. The event $A \cup B \cup C$ is the event the outcome is odd or black or low. The event $A \cup B \cup C$ is:

{1, 2, 3, ... , 29, 31, 33, 35}

or

{All simple events except 00, 0, 30, 32, 34, 36}

h. $P(A \cup B \cup C) = 32\left(\frac{1}{38}\right) = \frac{32}{38} = \frac{16}{19}$

3.51 a. The sample points for this experiment are:

(PTW-R, Jury), (PTW-R, Judge), (PTW-A/D, Jury), (PTW-A/D, Judge), (DTW-R, Jury), (DTW-R, Judge), (DTW-A/D, Jury), (DTW-A/D, Judge)

b. $P(A) = \frac{1{,}465}{2{,}143} = .684$

c. $P(B) = \frac{265}{2{,}143} = .124$

d. No. $P(A \cap B) = \frac{194}{2{,}143} = .091$. Since this is not 0, events A and B are not mutually exclusive.

e. $P(A^c) = 1 - \text{P(A)} = 1 - .684 = .316$.

f. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .684 + .124 - .091 = .717$

g. $P(A \cap B) = \frac{194}{2{,}143} = .091$

3.64 a. $P(A \cap B) = P(A \mid B)P(B) = .6(.2) = .12$

b. $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.12}{.4} = .3$

3.66 Since A, B, and C are all mutually exclusive, we know that

$P(A \cap B) = P(A \cap C) = P(B \cap C) = 0$

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .30 + .55 - 0 = .85$

b. $P(A \cap B) = 0$

c. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{.55} = 0$

d. $P(B \cup C) = P(B) + P(C) - P(B \cap C) = .55 + .15 - 0 = .70$

e. No, B and C are not independent events. If B and C are independent events, then $P(B \mid C) = P(B)$. From the problem, we know $P(B) = .55$.

$P(B \mid C) = \dfrac{P(B \cap C)}{P(C)} = \dfrac{0}{.15} = 0$. Thus, since $P(B \mid C) \neq P(B)$, events B and C are not independent.

3.69 The 36 possible outcomes obtained when tossing two dice are listed below:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

$A$: {(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 4), (5, 6), (6, 1), (6, 3), (6, 5)}

$B$: {(3, 6), (4, 5), (5, 4), (5, 6), (6, 3), (6, 5), (6, 6)}

$A \cap B$: {(3, 6), (4, 5), (5, 4), (5, 6), (6, 3), (6, 5)}

If $A$ and $B$ are independent, then $P(A)P(B) = P(A \cap B)$.

$P(A) = \dfrac{18}{36} = \dfrac{1}{2}$ $\quad P(B) = \dfrac{7}{36}$ $\quad P(A \cap B) = \dfrac{6}{36} = \dfrac{1}{6}$

$P(A)P(B) = \dfrac{1}{2} \cdot \dfrac{7}{36} = \dfrac{7}{72} \neq \dfrac{1}{6} = P(A \cap B)$. Thus, $A$ and $B$ are not independent.

3.71 Let $W_1$ and $W_2$ represent the two white chips, $R_1$ and $R_2$ represent the two red chips, and $B_1$ and $B_2$ represent the two blue chips. The sample space is:

| | | |
|---|---|---|
| $W_1W_2$ | $W_2R_1$ | $R_1B_1$ |
| $W_1R_1$ | $W_2R_2$ | $R_1B_2$ |
| $W_1R_2$ | $W_2B_1$ | $R_2B_1$ |
| $W_1B_1$ | $W_2B_2$ | $R_2B_2$ |
| $W_1B_2$ | $R_1R_2$ | $B_1B_2$ |

Assuming each event is equally likely, each event will have a probability of 1/15.

Then, $P(A) = P(W_1W_2) + P(R_1R_2) + P(B_1B_2) = 3\left(\frac{1}{15}\right) = \frac{3}{15} = \frac{1}{5}$

$$P(B) = P(R_1R_2) = \frac{1}{15}$$

$$\begin{aligned} P(C) &= P(W_1W_2) + P(W_1R_1) + P(W_1R_2) + P(W_1B_1) + P(W_1B_2) + P(W_2R_1) \\ &\quad + P(W_2R_2) + P(W_2B_1) + P(W_2B_2) + P(R_1R_2) + P(R_1B_1) + P(R_1B_2) \\ &\quad + P(R_2B_1) + P(R_2B_2) \\ &= 14\left(\frac{1}{15}\right) = \frac{14}{15} \end{aligned}$$

$$P(A \cap B) = P(R_1R_2) = \frac{1}{15}$$

$$P(A^c) = 1 - P(A) = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P(A^c \cap B) = 0$$

$$P(B \cap C) = P(R_1R_2) = \frac{1}{15}$$

$$P(A \cap C) = P(W_1W_2) + P(R_1R_2) = 2\left(\frac{1}{15}\right) = \frac{2}{15}$$

$$\begin{aligned} P(A^c \cap C) &= P(W_1R_1) + P(W_1R_2) + P(W_1B_1) + P(W_1B_2) + P(W_2R_1) \\ &\quad + P(W_2R_2) + P(W_2B_1) + P(W_2B_2) + P(R_1B_1) + P(R_1B_2) \\ &\quad + P(R_2B_1) + P(R_2B_2) \\ &= 12\left(\frac{1}{15}\right) = \frac{12}{15} = \frac{4}{5} \end{aligned}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{15}}{\frac{1}{5}} = \frac{1}{3}$$

$$P(B \mid A^c) = \frac{P(A^c \cap B)}{P(A^c)} = \frac{0}{\frac{4}{5}} = 0$$

$$P(B \mid C) = \frac{P(B \cap C)}{P(C)} = \frac{\frac{1}{15}}{\frac{14}{15}} = \frac{1}{14}$$

$$P(A \mid C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{2}{15}}{\frac{14}{15}} = \frac{1}{7}$$

$$P(C \mid A^c) = \frac{P(A^c \cap C)}{P(A^c)} = \frac{\frac{4}{5}}{\frac{4}{5}} = 1$$

3.72 Define the following events:

$A$: {Child has neuroblastoma}

$B$: {Child undergoes surgery}

$C$: {Surgery is successful in curing the disease}

From the problem, we know $P(B \mid A) = .20$ and $P(C \mid B \cap A) = .95$.

We also know that $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$ and $P(C \mid B \cap A) = \dfrac{P(C \cap B \cap A)}{P(B \cap A)}$

Thus, $P(C \cap B \mid A) = \dfrac{P(C \cap B \cap A)}{P(A)} = \dfrac{P(C \cap B \cap A)}{P(B \cap A)} \dfrac{P(B \cap A)}{P(A)} = P(C \mid B \cap A)P(B|A)$

$= .95(.20) = .19$

3.73 Let $A$ = {Executive cheated at golf} and $B$ = {Executive lied in business}. From the problem, $P(A) = .55$ and $P(A \cap B) = .20$.

$P(B \mid A) = P(A \cap B) / P(A) = .20 / .55 = .364$

3.77 Define the following events:

A: {Winner is from National League}

B: {Winner is from American League}

C: {Winner is from Eastern Division}

D: {Winner is from Central Division}

E: {Winner is from Western Division}

a. $P(C \mid B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{6/13}{8/13} = \dfrac{6}{8} = .75$

b. $P(A \mid D) = \dfrac{P(A \cap D)}{P(D)} = \dfrac{1/13}{2/13} = \dfrac{1}{2} = .5$

c. $P(C^c \mid A) = \dfrac{P(C^c \cap A)}{P(A)} = \dfrac{2/13}{5/13} = \dfrac{2}{5} = .4$