STAT 213 L05

Solutions to Assignment #6

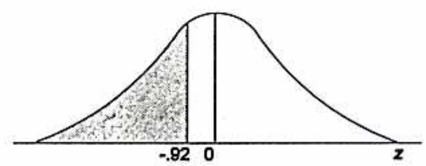
(Basis for Quiz #5 on Dec 5/7).

5.71 x is a binomial random variable with $n = 100$ and $p = .4$.

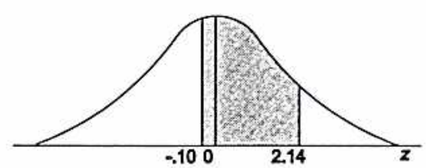
$$\mu \pm 3\sigma \Rightarrow np \pm 3\sqrt{npq} \Rightarrow 100(.4) \pm 3\sqrt{100(.4)(1-.4)}$$
$$\Rightarrow 40 \pm 3(4.8990) \Rightarrow (25.303, 54.697)$$

Since the interval lies in the range 0 to 100, we can use the normal approximation to approximate the probabilities.

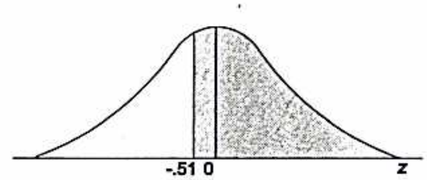
a. $P(x \le 35) \approx P\left(z \le \dfrac{(35+.5)-40}{4.899}\right)$
$= P(z \le -.92)$
$= .5000 - .3212 = .1788$
(Using Table IV in Appendix A.)



b. $P(40 \le x \le 50)$
$\approx P\left(\dfrac{(40-.5)-40}{4.899} \le z \le \dfrac{(50+.5)-40}{4.899}\right)$
$= P(-.10 \le z \le 2.14)$
$= P(-.10 \le z \le 0) + P(0 \le z \le 2.14)$
$= .0398 + .4838 = .5236$
(Using Table IV in Appendix A.)



c. $P(x \ge 38) \approx P\left(z \ge \dfrac{(38-.5)-40}{4.899}\right)$
$= P(z \ge -.51)$
$= .5000 + .1950 = .6950$
(Using Table IV in Appendix A.)



5.81 a. Let x = number of abused women in a sample of 150. The random variable x is a binomial random variable with $n = 150$ and $p = 1/3$. Thus, for the normal approximation,

$$\mu = np = 150(1/3) = 50 \text{ and } \sigma = \sqrt{npq} = \sqrt{150(1/3)(2/3)} = 5.7735$$
$$\mu \pm 3\sigma \Rightarrow 50 \pm 3(5.7735) \Rightarrow 50 \pm 17.3205 \Rightarrow (32.6795, 67.3205)$$

Since this interval lies in the range from 0 to $n = 150$, the normal approximation is appropriate.

$$P(x > 75) \approx P\left(z > \frac{(75+.5)-50}{5.7735}\right) = P(z > 4.42) \approx .5 - .5 = 0$$

(Using Table IV, Appendix A.)

b. $$P(x < 50) \approx P\left(z < \frac{(50-.5)-50}{5.7735}\right) = P(z < -.09) \approx .5 - .0359 = .4641$$

c. $$P(x < 30) \approx P\left(z < \frac{(30-.5)-50}{5.7735}\right) = P(z < -3.55) \approx .5 - .5 = 0$$

Since the probability of seeing fewer than 30 abused women in a sample of 150 is so small ($p \approx 0$), it would be very unlikely to see this event.

7.10 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.96\frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .56 \Rightarrow (25.34, 26.46)$$

b. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The confidence interval is:

$$\bar{x} \pm z_{.05}\frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.645\frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .47 \Rightarrow (25.43, 26.37)$$

c. For confidence coefficient .99, $\alpha = .01$ and $\alpha/2 = .01/2 = .005$. From Table IV, Appendix A, $z_{.005} = 2.58$. The confidence interval is:

$$\bar{x} \pm z_{.005}\frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 2.58\frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .73 \Rightarrow (25.17, 26.63)$$

7.12 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} \Rightarrow 33.9 \pm 1.96\frac{3.3}{\sqrt{100}} \Rightarrow 33.9 \pm .647 \Rightarrow (33.253, 34.547)$$

b. $$\bar{x} \pm z_{.025}\frac{s}{\sqrt{n}} \Rightarrow 33.9 \pm 1.96\frac{3.3}{\sqrt{400}} \Rightarrow 33.9 \pm .323 \Rightarrow (33.577, 34.223)$$

c. For part **a**, the width of the interval is 2(.647) = 1.294. For part **b**, the width of the interval is 2(.323) = .646. When the sample size is quadrupled, the width of the confidence interval is halved.

7.14 a. The point estimate for the mean personal network size of all older adults is $\bar{x} = 14.6$.

b. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The 95% confidence interval is:

$$\bar{x} \pm z_{.025}\sigma_{\bar{x}} \Rightarrow \bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}} \Rightarrow 14.6 \pm 1.96\frac{9.8}{\sqrt{2{,}819}} \Rightarrow 14.6 \pm .36 \Rightarrow (14.24,\ \ 14.96)$$

c. We are 95% confident that the mean personal network size of all older adults is between 14.24 and 14.96.

d. We must assume that we have a random sample from the target population and that the sample size is sufficiently large.

7.18 a. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The confidence interval is:

$$\bar{x} \pm z_{.05}\frac{s}{\sqrt{n}} \Rightarrow 7.62 \pm 1.645\frac{8.91}{\sqrt{65}} \Rightarrow 7.62 \pm 1.82 \Rightarrow (5.80, 9.44)$$

b. We are 90% confident that the mean sentence complexity score of all low-income children is between 5.80 and 9.44.

c. Yes. We are 90% confident that the mean sentence complexity score of all low-income children is between 5.80 and 9.44. Since the mean score for middle-income children, 15.55, is outside this interval, there is evidence that the true mean for low-income children is different from 15.55.

5

7.30 a. $P(-t_0 < t < t_0) = .95$ where df = 16

Because of symmetry, the statement can be written

$$P(0 < t < t_0) = .475 \text{ where df} = 16$$

$$\Rightarrow P(t \geq t_0) = .5 - .475 = .025$$

$$t_0 = 2.120$$

b. $P(t \leq -t_0 \text{ or } t \geq t_0) = .05$ where df = 16

$$\Rightarrow 2P(t \geq t_0) = .05$$

$$\Rightarrow P(t \geq t_0) = .025 \text{ where df} = 16$$

$$t_0 = 2.120$$

c. $P(t \leq t_0) = .05$ where df = 16

Because of symmetry, the statement can be written

$$P(t \geq -t_0) = .05 \text{ where df} = 16$$

$$t_0 = -1.746$$

d. $P(t \leq -t_0 \text{ or } t \geq t_0) = .10$ where df = 12

$$\Rightarrow 2P(t \geq t_0) = .10$$

$$\Rightarrow P(t \geq t_0) = .05 \text{ where df} = 12$$

$$t_0 = 1.782$$

e. $P(t \leq -t_0 \text{ or } t \geq t_0) = .01$ where df = 8

$$\Rightarrow 2P(t \geq t_0) = .01$$

$$\Rightarrow P(t \geq t_0) = .005 \text{ where df} = 8$$

$$t_0 = 3.355$$

7.32 For this sample,

$$\bar{x} = \frac{\sum x}{n} = \frac{1567}{16} = 97.9375$$

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{155{,}867 - \frac{1567^2}{16}}{16-1} = 159.9292$$

$$s = \sqrt{s^2} = 12.6463$$

a. For confidence coefficient, .80, $\alpha = 1 - .80 = .20$ and $\alpha/2 = .20/2 = .10$. From Table VI, Appendix A, with df $= n - 1 = 16 - 1 = 15$, $t_{.10} = 1.341$. The 80% confidence interval for μ is:

$$\bar{x} \pm t_{.10}\frac{s}{\sqrt{n}} \Rightarrow 97.94 \pm 1.341\frac{12.6463}{\sqrt{16}} \Rightarrow 97.94 \pm 4.240 \Rightarrow (93.700, 102.180)$$

(continued)

7.32 (continued)

b. For confidence coefficient, .95, $\alpha = 1 - .95 = .05$ and $\alpha/2 = .05/2 = .025$. From Table VI, Appendix A, with df = $n - 1 = 24 - 1 = 23$, $t_{.025} = 2.131$. The 95% confidence interval for μ is:

$$\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}} \Rightarrow 97.94 \pm 2.131\frac{12.6463}{\sqrt{16}} \Rightarrow 97.94 \pm 6.737 \Rightarrow (91.203,\ 104.677)$$

The 95% confidence interval for μ is wider than the 80% confidence interval for μ found in part **a**.

c. For part **a**:

We are 80% confident that the true population mean lies in the interval 93.700 to 102.180.

For part **b**:

We are 95% confident that the true population mean lies in the interval 91.203 to 104.677.

The 95% confidence interval is wider than the 80% confidence interval because the more confident you want to be that μ lies in an interval, the wider the range of possible values.

7.34 a. The point estimate for the average annual rainfall amount at ant sites in the Dry Steppe region of Central Asia is $\bar{x} = 183.4$ milliliters.

b. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table VI, Appendix A, with df = $n - 1 = 5 - 1 = 4$, $t_{.05} = 2.132$.

c. The 90% confidence interval is:

$$\bar{x} \pm t_{.05}\frac{s}{\sqrt{n}} \Rightarrow 183.4 \pm 2.132\frac{20.6470}{\sqrt{5}} \Rightarrow 183.4 \pm 19.686 \Rightarrow (163.714,\ \ 203.086)$$

d. We are 90% confident that the average annual rainfall amount at ant sites in the Dry Steppe region of Central Asia is between 163.714 and 203.086 milliliters.

e. Using MINITAB, the 90% confidence interval is:

One-Sample T: DS Rain

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Variable  N     Mean   StDev  SE Mean        90% CI
DS Rain   5  183.400  20.647    9.234  (163.715, 203.085)
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The 90% confidence interval is (163.715, 203.085). This is very similar to the confidence interval calculated in part c.

(continued)

7.39 (continued)

f. The point estimate for the average annual rainfall amount at ant sites in the Gobi Desert region of Central Asia is $\bar{x} = 110.0$ milliliters.

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table VI, Appendix A, with df $= n - 1 = 6 - 1 = 5$, $t_{.05} = 2.015$.

The 90% confidence interval is:

$$\bar{x} \pm t_{.05}\frac{s}{\sqrt{n}} \Rightarrow 110.0 \pm 2.015\frac{15.975}{\sqrt{6}} \Rightarrow 110.0 \pm 13.141 \Rightarrow (96.859,\ 123.141)$$

We are 90% confident that the average annual rainfall amount at ant sites in the Gobi Desert region of Central Asia is between 96.859 and 123.141 milliliters.

Using MINITAB, the 90% confidence interval is:

One-Sample T: GD Rain

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Variable  N     Mean   StDev  SE Mean        90% CI
GD Rain   6  110.000  15.975    6.522  (96.858, 123.142)
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The 90% confidence interval is (96.858, 123.142). This is very similar to the confidence interval calculated above.

7.48 a. Of the 50 observations, 15 like the product $\Rightarrow \hat{p} = \frac{15}{30} = .30$.

To see if the sample size is sufficiently large:

$$\hat{p} \pm 3\sigma_{\hat{p}} \approx \hat{p} \pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .3 \pm 3\sqrt{\frac{.3(.7)}{50}} \Rightarrow .3 \pm .194 \Rightarrow (.106, .494)$$

Since this interval is wholly contained in the interval (0, 1), we may conclude that the normal approximation is reasonable.

For the confidence coefficient .80, $\alpha = .20$ and $\alpha/2 = .10$. From Table IV, Appendix A, $z_{.10} = 1.28$. The confidence interval is:

$$\hat{p} \pm z_{.10}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .3 \pm 1.28\sqrt{\frac{.3(.7)}{50}} \Rightarrow .3 \pm .083 \Rightarrow (.217, .383)$$

b. We are 80% confident the proportion of all consumers who like the new snack food is between .217 and .383.

7.50 a. The point estimate of p is $\hat{p} = x/n = 39/150 = .26$.

b. We must check to see if the sample size is sufficiently large:

$$\hat{p} \pm 3\sigma_{\hat{p}} \approx \hat{p} \pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .26 \pm 3\sqrt{\frac{.26(.74)}{150}} \Rightarrow .26 \pm .107 \Rightarrow (.153, .367)$$

Since the interval is wholly contained in the interval (0, 1), we may assume that the normal approximation is reasonable.

For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\hat{p} \pm z_{.025}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .26 \pm 1.96\sqrt{\frac{.26(.74)}{150}} \Rightarrow .26 \pm .070 \Rightarrow (.190, .330)$$

c. We are 95% confident that the true proportion of college students who experience "residual anxiety" from a scarey TV show or movie is between .190 and .330.

7.54 a. The population of interest to the Gallup Organization is the set of all debit cardholders in the U.S.

b. First, we compute $\hat{p}$: $\hat{p} = \frac{x}{n} = \frac{180}{1252} = .144$

To see if the sample size is sufficiently large:

$$\hat{p} \pm 3\sigma_{\hat{p}} \approx \hat{p} \pm 3\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .144 \pm 3\sqrt{\frac{.144(.856)}{1252}} \Rightarrow .144 \pm .030 \Rightarrow (.114, .174)$$

Since the interval is wholly contained in the interval (0,1), we may conclude that the normal approximation is reasonable.

c. For confidence coefficient .99, $\alpha = .01$ and $\alpha/2 = .01/2 = .005$. From Table IV, Appendix A, $z_{.005} = 2.58$. The confidence interval is:

$$\hat{p} \pm z_{.005}\sqrt{\frac{pq}{n}} \approx \hat{p} \pm 2.58\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .144 \pm 2.58\sqrt{\frac{.144(.856)}{1252}} \Rightarrow .144 \pm .026 \Rightarrow (.118, .170)$$

We are 99% confident that the proportion of debit cardholders who have used their card in making purchases over the internet is between .118 and .170.

d. A 90% confidence interval would be narrower. With less confidence, the range of possible values for the true proportion would have to be smaller.

7.66 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$.

The sample size is $n = \dfrac{(z_{\alpha/2})^2 pq}{(SE)^2} = \dfrac{(1.96)^2(.3)(.7)}{.06^2} = 224.1 \approx 225$

You would need to take $n = 225$ samples.

b. To compute the needed sample size, use:

$$n = \frac{(z_{\alpha/2})^2 pq}{(SE)^2} = \frac{(1.96)^2(.5)(.5)}{.06^2} = 266.8 \approx 267$$

You would need to take $n = 267$ samples.

7.70 a. The confidence level desired by the researchers is .95.

b. The sampling error desired by the researchers is $SE = .001$.

c. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$.

The sample size is $n = \dfrac{(z_{.025})^2\sigma^2}{(SE)^2} = \dfrac{1.96^2(.005)^2}{.001^2} = 96.04 \approx 97$.

7.72 For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. Since we have no estimate given for the value of p, we will use .5. The confidence interval is:

$$n = \frac{z_{\alpha/2}^2 pq}{(SE)^2} = \frac{1.645^2 .5(.5)}{.02^2} = 1{,}691.3 \approx 1{,}692$$

7.74 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table VI, Appendix A, with df $= n - 1 = 15 - 1 = 14$, $t_{.025} = 2.145$. The confidence interval is:

$$\bar{x} \pm t_{.025}\frac{s}{\sqrt{n}} \Rightarrow 5.87 \pm 2.145\frac{1.51}{\sqrt{15}} \pm 5.87 \pm .836 \Rightarrow (5.034, 6.706)$$

We are 95% confident that the true mean response of the students is between 5.034 and 6.706.

b. In part a, the width of the interval is $6.706 - 5.034 = 1.672$. The value of SE is $1.672/2 = .836$. If we want the interval to be half as wide, the value of SE would be half that in part a or $.836/2 = .418$. The necessary sample size is:

$$n = \frac{z_{\alpha/2}^2 \sigma^2}{(SE)^2} = \frac{1.96^2 1.51^2}{.418^2} = 50.13 \approx 51$$

7.76 a. The confidence interval might lead to an erroneous inference because it is so wide. The width of the interval is .459, which is very wide for making inferences.

b. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. From the previous study, we will use $\hat{p} = 10/18 = .5555$ to estimate p.

$$n = \frac{z_{\alpha/2}^2 pq}{(SE)^2} = \frac{1.96^2(.5555)(.4445)}{.04^2} = 592.85 \approx 593$$