

STAT 213 LOS

Solutions to Assignment #5

NOTE: Problems involving the Normal approximation to the Binomial distribution will not be on Quiz #4 (Nov 21/23).

5.3

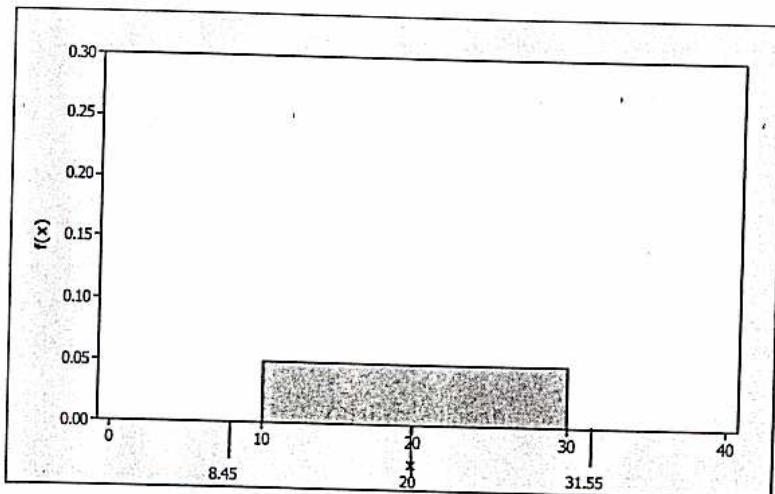
- a. For a uniform random variable,

$$f(x) = \begin{cases} \frac{1}{d-c} = \frac{1}{30-10} = \frac{1}{20} & 10 \leq x \leq 30 \\ 0 & \text{otherwise} \end{cases}$$

b. $\mu = \frac{c+d}{2} = \frac{10+30}{2} = 20$

$$\sigma = \frac{d-c}{\sqrt{12}} = \frac{30-10}{\sqrt{12}} = 5.774$$

c.



$$\mu \pm 2\sigma \Rightarrow 20 \pm 2(5.774) \Rightarrow 20 \pm 11.584 \Rightarrow (8.452, 31.548)$$

5.4

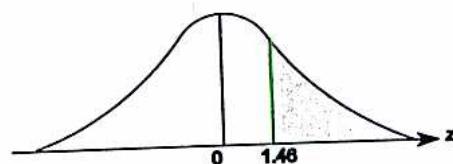
From Exercise 5.3, $f(x) = .05 \quad (10 \leq x \leq 30)$

- $P(10 \leq x \leq 25) = (25 - 10)(.05) = .75$
- $P(20 < x < 30) = (30 - 20)(.05) = .5$
- $P(x \geq 25) = (30 - 25)(.05) = .25$
- $P(x \leq 10) = (10 - 10)(.05) = 0$
- $P(x \leq 25) = (25 - 10)(.05) = .75$
- $P(20.5 \leq x \leq 25.5) = (25.5 - 20.5)(.05) = .25$

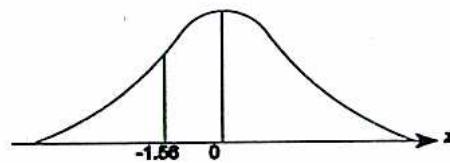
5.22

Using Table IV, Appendix A:

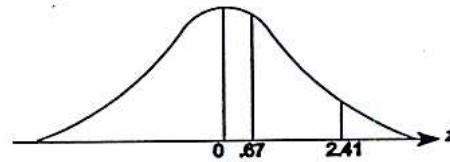
a. $P(z > 1.46) = .5 \quad P(0 < z \leq 1.46)$
 $= .5 - .4279 = .0721$



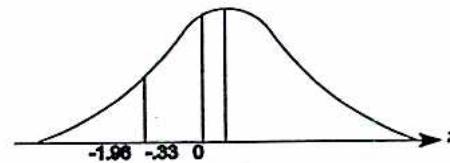
b. $P(x < -1.56) = .5 - P(-1.56 \leq z < 0)$
 $= .5 - .4406 = .0594$



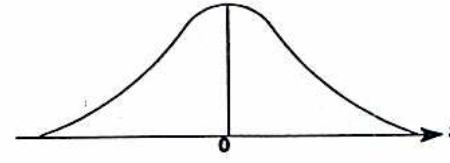
c. $P(.67 \leq z \leq 2.41)$
 $= P(0 < z \leq 2.41) - P(0 < z < .67)$
 $= .4920 - .2486 = .2434$



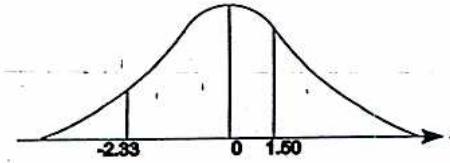
d. $P(-1.96 \leq z < -.33)$
 $= P(-1.96 \leq z < 0) - P(-.33 \leq z < 0)$
 $= .4750 - .1293 = .3457$



e. $P(z \geq 0) = .5$



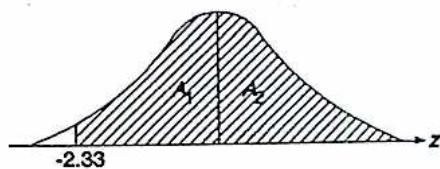
f. $P(-2.33 < z < 1.50)$
 $= P(-2.33 < z < 0) + P(0 < z < 1.50)$
 $= .4901 + .4332 = .9233$



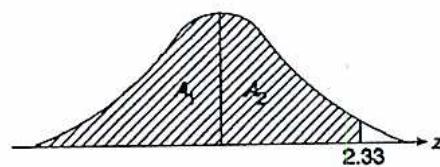
(continued)

5.22 (continued)

g. $P(z \geq -2.33) = P(-2.33 \leq z \leq 0) + P(z \geq 0)$
 $= .4901 + .5000$
 $= .9901$



h. $P(z < 2.33) = P(z \leq 0) + P(0 \leq z \leq 2.33)$
 $= .5000 + .4901$
 $= .9901$



5.26

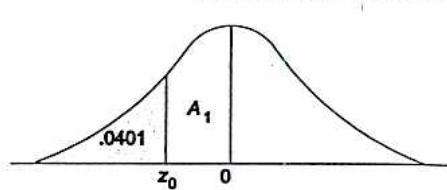
Using Table IV of Appendix A:

a. $P(z \leq z_0) = .0401$

$A_1 = .5000 - .0401 = .4591$

Look up the area $.4591$ in the body of Table IV;
 $z_0 = -1.75$

(z_0 is negative since the graph shows z_0 is on the left side of 0.)

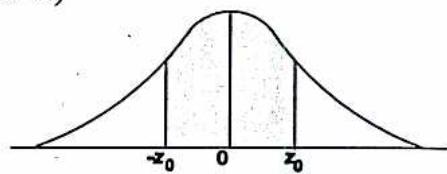


b. $P(-z_0 \leq z \leq z_0) = .95$

$P(-z_0 \leq z \leq z_0) = 2P(0 \leq z \leq z_0)$

$2P(0 \leq z \leq z_0) = .95$

Therefore, $P(0 \leq z \leq z_0) = .4750$

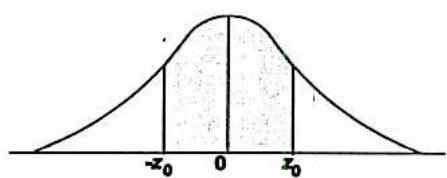


c. $P(-z_0 \leq z < z_0) = .90$

$P(-z_0 \leq z < z_0) = 2P(0 \leq z < z_0)$

$2P(0 \leq z < z_0) = .90$

Therefore, $P(0 \leq z \leq z_0) = .45$



Look up the area $.45$ in the body of Table IV; $z_0 = 1.645$ (.45 is half way between $.4495$

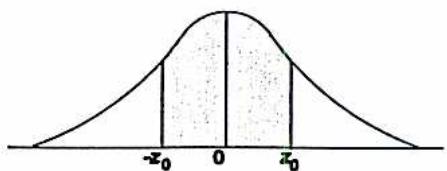
and $.4505$; therefore, we average the z -scores $\frac{1.64 + 1.65}{2} = 1.645$)

d. $P(-z_0 \leq z \leq z_0) = .8740$

$P(-z_0 \leq z \leq z_0) = 2P(0 \leq z \leq z_0)$

$2P(0 \leq z \leq z_0) = .8740$

Therefore, $P(0 \leq z \leq z_0) = .4370$



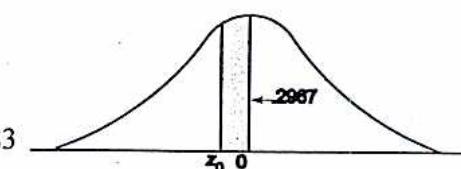
Look up the area $.4370$ in the body of Table IV; $z_0 = 1.53$

5.28 (continued)

e. $P(z_0 \leq z \leq 0) = .2967$

$$P(z_0 \leq z \leq 0) = P(0 \leq z \leq -z_0)$$

Look up the area .2967 in the body of Table IV; $z_0 = -.83$

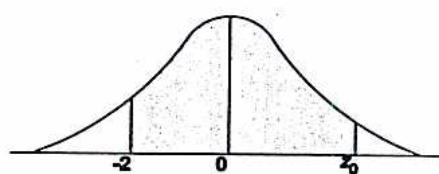


f. $P(-2 < z < z_0) = .9710$

$$P(-2 < z < z_0)$$

$$= P(-2 < z < 0) + P(0 < z < z_0) = .9710$$

$$P(0 < z < 2) + P(0 < z < z_0) = .9710$$

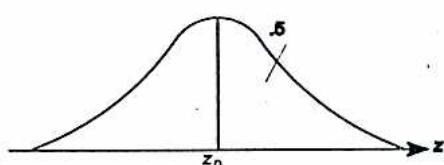


$$\text{Thus, } P(0 < z < z_0) = .9710 - P(0 < z < 2) = .9710 - .4772 = .4938$$

Look up the area .4938 in the body of Table IV; $z_0 = 2.50$

g. $P(z \geq z_0) = .5$

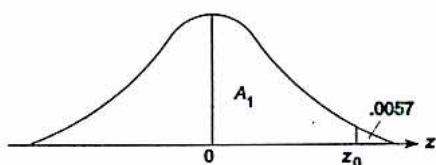
$$z_0 = 0$$



h. $P(z \geq z_0) = .0057$

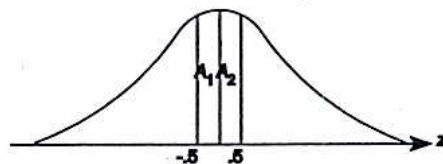
$$A_1 = .5 - .0057 = .4943$$

Looking up the area .4943 in Table IV gives $z_0 = 2.53$.

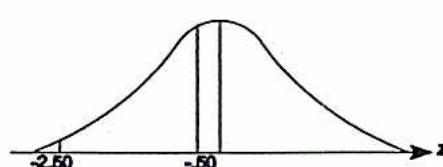


5.28

a. $P(10 \leq x \leq 12) = P\left(\frac{10-11}{2} \leq z \leq \frac{12-11}{2}\right)$
 $= P(-0.50 \leq z \leq 0.50)$
 $= A_1 + A_2$
 $= .1915 + .1915 = .3830$



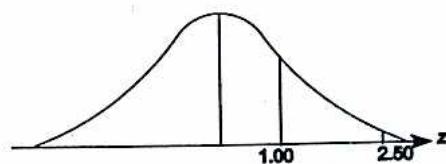
b. $P(6 \leq x \leq 10) = P\left(\frac{6-11}{2} \leq z \leq \frac{10-11}{2}\right)$
 $= P(-2.50 \leq z \leq -0.50)$
 $= P(-2.50 \leq z \leq 0)$
 $- P(-0.50 \leq z \leq 0)$
 $= .4938 - .1915 = .3023$



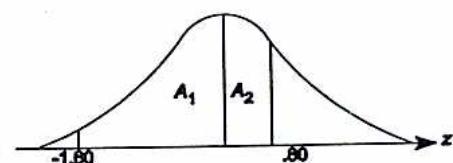
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(continued)

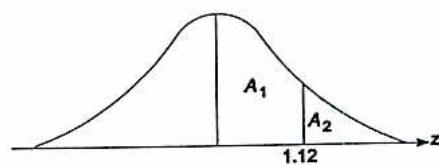
$$\begin{aligned}
 \text{c. } P(13 \leq x \leq 16) &= P\left(\frac{13-11}{2} \leq z \leq \frac{16-11}{2}\right) \\
 &= P(1.00 \leq z \leq 2.50) \\
 &= P(0 \leq z \leq 2.50) \\
 &\quad - P(0 \leq z \leq 1.00) \\
 &= .4938 - .3413 = .1525
 \end{aligned}$$



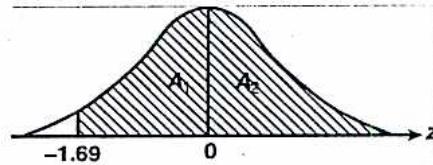
$$\begin{aligned}
 \text{d. } P(7.8 \leq x \leq 12.6) &= P\left(\frac{7.8-11}{2} \leq z \leq \frac{12.6-11}{2}\right) \\
 &= P(-1.60 \leq z \leq 0.80) \\
 &= A_1 + A_2 \\
 &= .4452 + .2881 = .7333
 \end{aligned}$$



$$\begin{aligned}
 \text{e. } P(x \geq 13.24) &= P\left(z \geq \frac{13.24-11}{2}\right) \\
 &= P(z \geq 1.12) \\
 &= A_2 = .5 - A_1 \\
 &= .5000 - .3686 = .1314
 \end{aligned}$$



$$\begin{aligned}
 \text{f. } P(x \geq 7.62) &= P\left(z \geq \frac{7.62-11}{2}\right) \\
 &= P(z \geq -1.69) \\
 &= A_1 + A_2 \\
 &= .4545 + .5000 = .9545
 \end{aligned}$$



5.34

a. Let x = score on Dental Anxiety Scale. Then $z = \frac{x-\mu}{\sigma} = \frac{16-11}{3.5} = 1.43$

b. Using Table IV, Appendix A,

$$\begin{aligned}
 P(10 < x < 15) &= P\left(\frac{10-11}{3.5} < z < \frac{15-11}{3.5}\right) = P(-.29 < z < 1.14) \\
 &= P(-.29 < z < 0) + P(0 < z < 1.14) = .1141 + .3729 = .4870
 \end{aligned}$$

c. Using Table IV, Appendix A,

$$P(x > 17) = P\left(z > \frac{17-11}{3.5}\right) = P(z > 1.71) = .5 - P(0 < z < 1.71) = .5 - .4564 = .0436$$

5.36

- a. Let x = change in SAT-MATH score. Using Table IV, Appendix A,

$$P(x \geq 50) = P\left(z \geq \frac{50-19}{65}\right) = P(z \geq .48) = .5 - .1844 = .3156.$$

- b. Let x = change in SAT-VERBAL score. Using Table IV, Appendix A,

$$P(x \geq 50) = P\left(z \geq \frac{50-7}{49}\right) = P(z \geq .88) = .5 - .3106 = .1894.$$

5.38

- a. Let x = weight of captured fish. Using Table IV, Appendix A,

$$\begin{aligned} P(1,000 < x < 1,400) &= P\left(\frac{1,000-1,050}{375} < z < \frac{1,400-1,050}{375}\right) = P(-.13 < z < .93) \\ &= .0517 + .3238 = .3755 \end{aligned}$$

$$\begin{aligned} b. \quad P(800 < x < 1,000) &= P\left(\frac{800-1,050}{375} < z < \frac{1,000-1,050}{375}\right) = P(-.67 < z < -.13) \\ &= .2486 - .0517 = .1969 \end{aligned}$$

$$c. \quad P(x < 1,750) = P\left(z < \frac{1,750-1,050}{375}\right) = P(z < 1.87) = .5 + .4693 = .9693$$

$$d. \quad P(x > 500) = P\left(z > \frac{500-1,050}{375}\right) = P(z > -1.47) = .5 + .4292 = .9292$$

$$\begin{aligned} e. \quad P(x < x_o) = .95 &\Rightarrow P\left(z < \frac{x_o-1,050}{375}\right) = .95 \Rightarrow z = 1.645 = \frac{x_o-1,050}{375} \\ &\Rightarrow 616.875 = x_o - 1,050 \Rightarrow x_o = 1,666.875 \end{aligned}$$

5.70

- a. Using Table II, $P(x \leq 11) = .345$

$$\mu = np = 25(.5) = 12.5, \sigma = \sqrt{npq} = \sqrt{25(.5)(.5)} = 2.5$$

- Using the normal approximation,

$$P(x \leq 11) \approx P\left(z \leq \frac{(11+.5)-12.5}{2.5}\right) = P(z \leq -.40) = .5 - .1554 = .3446$$

- b. Using Table II, $P(x \geq 16) = 1 - P(x \leq 15) = 1 - .885 = .115$

- Using the normal approximation,

$$P(x \geq 16) \approx P\left(z \geq \frac{(16-.5)-12.5}{2.5}\right) = P(z \geq 1.2) = .5 - .3849 = .1151$$

(from Table IV, Appendix A)

(continued)

(5.70) (continued)

- c. Using Table II, $P(8 \leq x \leq 16) = P(x \leq 16) - P(x \leq 7) = .946 - .022 = .924$

Using the normal approximation,

$$\begin{aligned} P(8 \leq x \leq 16) &\approx P\left(\frac{(8-.5)-12.5}{2.5} \leq z \leq \frac{(16+.5)-12.5}{2.5}\right) \\ &= P(-2.0 \leq z \leq 1.6) = .4772 + .4452 = .9224 \\ &\quad (\text{from Table IV, Appendix A}) \end{aligned}$$

5.72

$$\mu = np = 1000(.5) = 500, \sigma = \sqrt{npq} = \sqrt{1000(.5)(.5)} = 15.811$$

- a. Using the normal approximation,

$$\begin{aligned} P(x > 500) &\approx P\left(z > \frac{(500+.5)-500}{15.811}\right) = P(z > .03) = .5 - .0120 = .4880 \\ &\quad (\text{from Table IV, Appendix A}) \end{aligned}$$

$$\begin{aligned} b. P(490 \leq x < 500) &\approx P\left(\frac{(490-.5)-500}{15.811} \leq z < \frac{(500-.5)-500}{15.811}\right) \\ &= P(-.66 \leq z < -.03) = .2454 - .0120 = .2334 \\ &\quad (\text{from Table IV, Appendix A}) \end{aligned}$$

$$\begin{aligned} c. P(x > 550) &\approx P\left(z > \frac{(500+.5)-500}{15.811}\right) = P(z > 3.19) \approx .5 - .5 = 0 \\ &\quad (\text{from Table IV, Appendix A}) \end{aligned}$$

5.74

- a. $n = 10,000$ and $p = .15$

$$E(x) = \mu = np = 10,000(.15) = 1,500$$

$$\sigma^2 = npq = 10,000(.15)(.85) = 1,275$$

$$b. \sigma = \sqrt{1,275} = 35.707$$

Using the normal approximation to the binomial,

$$\begin{aligned} P(x > 1,600) &\approx P\left(z > \frac{(1,600+.5)-1,500}{35.707}\right) = P(z > 2.81) = .5 - .4975 = .0025 \\ &\quad (\text{from Table IV, Appendix A}) \end{aligned}$$

- c. Using the normal approximation to the binomial,

$$\begin{aligned} P(x > 6,500) &\approx P\left(z > \frac{(6,500+.5)-1,500}{35.707}\right) = P(z > 140.04) \approx .5 - .5 = 0 \\ &\quad (\text{from Table IV, Appendix A}) \end{aligned}$$

Since this probability is essentially 0, we would not expect to see more than 6,500 deaths in any one year.

5.76

- a. $\mu = E(x) = np = 1000(0.22) = 220$. This is the same value that was found in Exercise 4.58 a.
- b. $\sigma = \sqrt{npq} = \sqrt{1000(0.22)(1-0.22)} = \sqrt{171.6} = 13.10$. This is the same value that was found in Exercise 4.58 b.

$$c. z = \frac{x - \mu}{\sigma} = \frac{200.5 - 220}{13.10} = -1.49$$

$$d. P(x \leq 200) \approx P\left(z \leq \frac{200.5 - 220}{13.10}\right) = P(z \leq -1.49) = .5 - .4319 = .0681$$

(Using Table IV, Appendix A)

5.78

Let x = number of parents who condone spanking in 150 trials. Then x is a binomial random variable with $n = 150$ and $p = .6$.

$$\mu = np = 150(.6) = 90$$

$$\sigma = \sqrt{npq} = \sqrt{150(.6)(.4)} = \sqrt{36} = 6$$

$$P(x \leq 20) \approx P\left(z \leq \frac{(20+.5)-90}{6}\right) = P(z \leq -11.58) \approx 0$$

If, in fact, 60% of parents with young children condone spanking, the probability of seeing no more than 20 out of 150 parent clients who condone spanking is essentially 0. Thus, the claim made by the psychologist is either incorrect or the 60% figure is too high.

6.5

If the observations are independent of each other, then

$$P(1, 1) = p(1)p(1) = .2(.2) = .04$$

$$P(1, 2) = p(1)p(2) = .2(.3) = .06$$

$$P(1, 3) = p(1)p(3) = .2(.2) = .04$$

etc.

a.

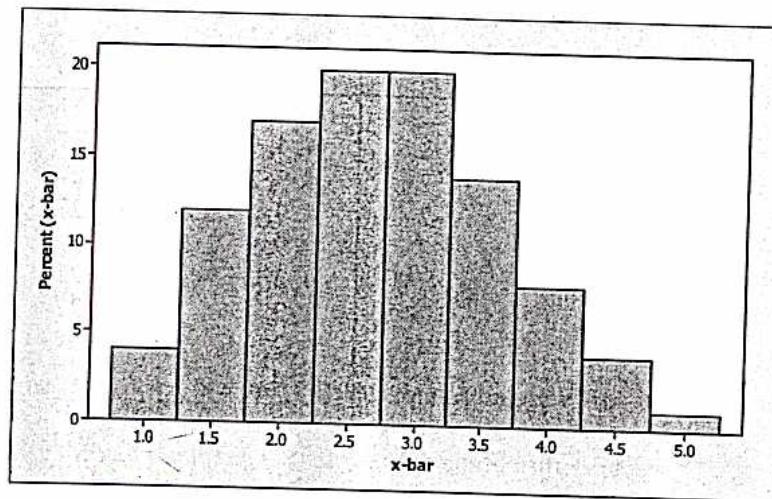
Possible Samples	\bar{x}	$p(\bar{x})$	Possible Samples	\bar{x}	$p(\bar{x})$
1, 1	1	.04	3, 4	3.5	.04
1, 2	1.5	.06	3, 5	4	.02
1, 3	2	.04	4, 1	2.5	.04
1, 4	2.5	.04	4, 2	3	.06
1, 5	3	.02	4, 3	3.5	.04
2, 1	1.5	.06	4, 4	4	.04
2, 2	2	.09	4, 5	4.5	.02
2, 3	2.5	.06	5, 1	3	.02
2, 4	3	.06	5, 2	3.5	.03
2, 5	3.5	.03	5, 3	4	.02
3, 1	2	.04	5, 4	4.5	.02
3, 2	2.5	.06	5, 5	5	.01
3, 3	3	.04			

(6.5) (continued)

Summing the probabilities, the probability distribution of \bar{x} is:

\bar{x}	$p(\bar{x})$
1	.04
1.5	.12
2	.17
2.5	.20
3	.20
3.5	.14
4	.08
4.5	.04
5	.01

b.



c. $P(\bar{x} \geq 4.5) = .04 + .01 = .05$

d. No. The probability of observing $\bar{x} = 4.5$ or larger is small (.05).

6.6

$$\begin{aligned} E(x) = \mu &= \sum xp(x) = 1(.2) + 2(.3) + 3(.2) + 4(.2) + 5(.1) \\ &= .2 + .6 + .6 + .8 + .5 = 2.7 \end{aligned}$$

$$\begin{aligned} E(\bar{x}) &= \sum \bar{x} p(\bar{x}) = 1.0(.04) + 1.5(.12) + 2.0(.17) + 2.5(.20) + 3.0(.20) + 3.5(.14) + 4.0(.08) \\ &\quad + 4.5(.04) + 5.0(.01) \\ &= .04 + .18 + .34 + .50 + .60 + .49 + .32 + .18 + .05 = 2.7 \end{aligned}$$

6.7

- a. For a sample of size $n = 2$, the sample mean and sample median are exactly the same. Thus, the sampling distribution of the sample median is the same as that for the sample mean (see Exercise 6.5a).
- b. The probability histogram for the sample median is identical to that for the sample mean (see Exercise 6.5b).

6.30

- a. $\mu_{\bar{x}} = \mu = 20, \sigma_{\bar{x}} = \sigma / \sqrt{n} = 16 / \sqrt{64} = 2$
- b. By the Central Limit Theorem, the distribution of \bar{x} is approximately normal. In order for the Central Limit Theorem to apply, n must be sufficiently large. For this problem, $n = 64$ is sufficiently large.
- c. $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{16 - 20}{2} = -2.00$
- d. $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{23 - 20}{2} = 1.50$
- e. $P(\bar{x} < 16) = P\left(z < \frac{16 - 20}{2}\right) = P(z < -2) = .5 - .4772 = .0228$
- f. $P(\bar{x} > 23) = P\left(z > \frac{23 - 20}{2}\right) = P(z > 1.50) = .5 - .4332 = .0668$
- g. $P(16 < \bar{x} < 22) = P\left(\frac{16 - 20}{2} < z < \frac{22 - 20}{2}\right) = P(-2 < z < 1)$
 $= .4772 + .3413 = .8185$

6.34

- a. $\mu_{\bar{x}}$ is the mean of the sampling distribution of \bar{x} . $\mu_{\bar{x}} = \mu = 106$.
- b. $\sigma_{\bar{x}}$ is the standard deviation of the sampling distribution of \bar{x} . $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16.4}{\sqrt{36}} = 2.73$
- c. By the Central Limit Theorem, the sampling distribution of \bar{x} is approximately normal.
- d. $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{100 - 106}{2.73} = -2.20$
- e. $P(\bar{x} < 100) = P(z < -2.20) = .5 - .4864 = .0136$ (Using Table IV, Appendix A.)

(1)

6.36

- a. Let \bar{x} = sample mean FNE score. By the Central Limit Theorem, the sampling distribution of \bar{x} is approximately normal with

$$\mu_{\bar{x}} = \mu = 18 \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{45}} = .7453.$$

$$P(\bar{x} > 17.5) = P\left(z > \frac{17.5 - 18}{.7453}\right) = P(z > -.67) = .5 + .2486 = .7486$$

(Using Table IV, Appendix A)

$$b. P(18 < \bar{x} < 18.5) = P\left(\frac{18 - 18}{.7453} < z < \frac{18.5 - 18}{.7453}\right) = P(0 < z < .67) = .2486$$

(Using Table IV, Appendix A)

$$c. P(\bar{x} < 18.5) = P\left(z < \frac{18.5 - 18}{.7453}\right) = P(z < .67) \Rightarrow .5 + .2486 = .7486$$

(Using Table IV, Appendix A)