

Solutions to Assignment #4

(basis for Quiz #3 on Nov. 7/9)

4.33

- a. It would seem that the mean of both would be 1 since they both are symmetric distributions centered at 1.
- b. $P(x)$ seems more variable since there appears to be greater probability for the two extreme values of 0 and 2 than there is in the distribution of y .

c. For x : $\mu = E(x) = \sum xp(x) = 0(.3) + 1(.4) + 2(.3) = 0 + .4 + .6 = 1$
 $\sigma^2 = E[(x - \mu)^2] = \sum (x - \mu)^2 p(x)$
 $= (0 - 1)^2(.3) + (1 - 1)^2(.4) + (2 - 1)^2(.3) = .3 + 0 + .3 = .6$

For y : $\mu = E(y) = \sum yp(y) = 0(.1) + 1(.8) + 2(.1) = 0 + .8 + .2 = 1$
 $\sigma^2 = E[(y - \mu)^2] = \sum (y - \mu)^2 p(y)$
 $= (0 - 1)^2(.1) + (1 - 1)^2(.8) + (2 - 1)^2(.1) = .1 + 0 + .1 = .2$

The variance for x is larger than that for y .

4.37

- a. For ARC a_1 : $\mu = E(x) = \sum xp(x) = 0(.05) + 1(.10) + 2(.25) + 3(.60) = 2.4$
 The mean capacity for ARC a_1 is 2.4

For ARC a_2 : $\mu = E(x) = \sum xp(x) = 0(.10) + 1(.30) + 2(.60) = 1.5$
 The mean capacity for ARC a_2 is 1.5

For ARC a_3 : $\mu = E(x) = \sum xp(x) = 0(.10) + 1(.90) = .90$
 The mean capacity for ARC a_3 is .90

For ARC a_4 : $\mu = E(x) = \sum xp(x) = 0(.10) + 1(.90) = .90$
 The mean capacity for ARC a_4 is .90

For ARC a_5 : $\mu = E(x) = \sum xp(x) = 0(.10) + 1(.90) = .90$
 The mean capacity for ARC a_5 is .90

For ARC a_6 : $\mu = E(x) = \sum xp(x) = 0(.05) + 1(.25) + 2(.70) = 1.65$

(continued)

4.37 (continued)

b. For ARC a_1 : $\sigma^2 = E(x - \mu)^2 = \sum (x - \mu)^2 p(x)$
 $= (3 - 2.4)^2(.60) + (2 - 2.4)^2(.25) + (1 - 2.4)^2(.10) + (0 - 2.4)^2(.05)$
 $= .216 + .040 + .196 + .288 = .74$
 $\sigma = \sqrt{\sigma^2} = \sqrt{.74} = .8602$

For ARC a_2 : $\sigma^2 = E(x - \mu)^2 = \sum (x - \mu)^2 p(x)$
 $= (2 - 1.5)^2(.60) + (1 - 1.5)^2(.30) + (0 - 1.5)^2(.10)$
 $= .15 + .075 + .225 = .45$
 $\sigma = \sqrt{\sigma^2} = \sqrt{.45} = .6708$

For ARC a_3 : $\sigma^2 = E(x - \mu)^2 = \sum (x - \mu)^2 p(x)$
 $= (1 - .9)^2(.90) + (0 - .9)^2(.10)$
 $= .009 + .081 = .09$
 $\sigma = \sqrt{\sigma^2} = \sqrt{.09} = .3$

For ARC a_4 : $\sigma^2 = E(x - \mu)^2 = \sum (x - \mu)^2 p(x)$
 $= (1 - .9)^2(.90) + (0 - .9)^2(.10)$
 $= .009 + .081 = .09$
 $\sigma = \sqrt{\sigma^2} = \sqrt{.09} = .3$

For ARC a_5 : $\sigma^2 = E(x - \mu)^2 = \sum (x - \mu)^2 p(x)$
 $= (1 - .9)^2(.90) + (0 - .9)^2(.10)$
 $= .009 + .081 = .09$
 $\sigma = \sqrt{\sigma^2} = \sqrt{.09} = .3$

For ARC a_6 : $\sigma^2 = E(x - \mu)^2 = \sum (x - \mu)^2 p(x)$
 $= (2 - 1.65)^2(.70) + (1 - 1.65)^2(.25) + (0 - 1.65)^2(.05)$
 $= .08575 + .105625 + .136125 = .3275$
 $\sigma = \sqrt{\sigma^2} = \sqrt{.3275} = .5723$

4.41

- a. Since there are 20 possible outcomes that are all equally likely, the probability of any of the 20 numbers is $1/20$. The probability distribution of x is:

$$P(x=5) = 1/20 = .05; P(x=10) = 1/20 = .05; \text{etc.}$$

x	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
$p(x)$.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05

$$\begin{aligned} \text{b. } E(x) &= \sum xp(x) = 5(.05) + 10(.05) + 15(.05) + 20(.05) + 25(.05) + 30(.05) + 35(.05) \\ &+ 40(.05) + 45(.05) + 50(.05) + 55(.05) + 60(.05) + 65(.05) + 70(.05) + 75(.05) \\ &+ 80(.05) + 85(.05) + 90(.05) + 95(.05) + 100(.05) = 52.5 \end{aligned}$$

$$\begin{aligned} \text{c. } \sigma^2 &= E(x - \mu)^2 = \sum (x - \mu)^2 p(x) = (5 - 52.5)^2(.05) + (10 - 52.5)^2(.05) \\ &+ (15 - 52.5)^2(.05) + (20 - 52.5)^2(.05) + (25 - 52.5)^2(.05) + (30 - 52.5)^2(.05) \\ &+ (35 - 52.5)^2(.05) + (40 - 52.5)^2(.05) + (45 - 52.5)^2(.05) + (50 - 52.5)^2(.05) \\ &+ (55 - 52.5)^2(.05) + (60 - 52.5)^2(.05) + (65 - 52.5)^2(.05) + (70 - 52.5)^2(.05) \\ &+ (75 - 52.5)^2(.05) + (80 - 52.5)^2(.05) + (85 - 52.5)^2(.05) + (90 - 52.5)^2(.05) \\ &+ (95 - 52.5)^2(.05) + (100 - 52.5)^2(.05) \\ &= 831.25 \end{aligned}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{831.25} = 28.83$$

Since the uniform distribution is not mound-shaped, we will use Chebyshev's theorem to describe the data. We know that at least $8/9$ of the observations will fall within 3 standard deviations of the mean and at least $3/4$ of the observations will fall within 2 standard deviations of the mean. For this problem,

$\mu \pm 2\sigma \Rightarrow 52.5 \pm 2(28.83) \Rightarrow 52.5 \pm 57.66 \Rightarrow (-5.16, 110.16)$. Thus, at least $3/4$ of the data will fall between -5.16 and 110.16 . For our problem, all of the observations will fall within 2 standard deviations of the mean. Thus, x is just as likely to fall within any interval of equal length.

- d. If a player spins the wheel twice, the total number of outcomes will be $20(20) = 400$. The sample space is:

5, 5	10, 5	15, 5	20, 5	25, 5...	100, 5
5, 10	10, 10	15, 10	20, 10	25, 10...	100, 10
5, 15	10, 15	15, 15	20, 15	25, 15...	100, 15
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
5, 100	10, 100	15, 100	20, 100	25, 100...	100, 100

Each of these outcomes are equally likely, so each has a probability of $1/400 = .0025$.

(continued)

4.41 (d) (continued)

Now, let x equal the sum of the two numbers in each sample. There is one sample with a sum of 10, two samples with a sum of 15, three samples with a sum of 20, etc. If the sum of the two numbers exceeds 100, then x is zero. The probability distribution of x is:

x	$p(x)$
0	.5250
10	.0025
15	.0050
20	.0075
25	.0100
30	.0125
35	.0150
40	.0175
45	.0200
50	.0225
55	.0250
60	.0275
65	.0300
70	.0325
75	.0350
80	.0375
85	.0400
90	.0425
95	.0450
100	.0475

e. We assumed that the wheel is fair, or that all outcomes are equally likely.

f.
$$\mu = E(x) = \sum xp(x) = 0(.5250) + 10(.0025) + 15(.0050) + 20(.0075) + \dots + 100(.0475) = 33.25$$

$$\begin{aligned}\sigma^2 = E(x - \mu)^2 &= \sum (x - \mu)^2 p(x) = (0 - 33.25)^2(.525) + (10 - 33.25)^2(.0025) \\ &\quad + (15 - 33.25)^2(.0050) + (20 - 33.25)^2(.0075) + \dots + (100 - 33.25)^2(.0475) \\ &= 1471.3125\end{aligned}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1471.3125} = 38.3577$$

g. $P(x = 0) = .525$

h. Given that the player obtains a 20 on the first spin, the possible values for x (sum of the two spins) are 0 (player spins 85, 90, 95, or 100 on the second spin), 25, 30, ..., 100. In order to get a sum of 25, the player would spin a 5 on the second spin. Similarly, the player would have to spin a 10 on the second spin order to get an x of 30, etc. Since all of the outcomes are equally likely on the second spin, the distribution of x is:

(continued)

7.41(b) (continued)

x	$p(x)$
0	.20
25	.05
30	.05
35	.05
40	.05
45	.05
50	.05
55	.05
60	.05
65	.05
70	.05
75	.05
80	.05
85	.05
90	.05
95	.05
100	.05

- i. The probability that the players total score will exceed one dollar is the probability that x is zero. $P(x = 0) = .20$

- j. Given that the player obtains a 65 on the first spin, the possible values for x (sum of the two spins) are 0 (player spins 40, 45, 50, up to 100 on second spin), 70, 75, 80, ..., 100. In order to get an x of 70, the player would spin a 5 on the second spin. Similarly, the player would have to spin a 10 on the second spin in order to get an x of 75, etc. Since all of the outcomes are equally likely on the second spin, the distribution of x is:

x	$p(x)$
0	.65
70	.05
75	.05
80	.05
85	.05
90	.05
95	.05
100	.05

The probability that the players total score will exceed one dollar is the probability that x is zero. $P(x = 0) = .65$.

4.51

a. $\mu = np = 25(.5) = 12.5$
 $\sigma^2 = np(1-p) = 25(.5)(.5) = 6.25$
 $\sigma = \sqrt{\sigma^2} = \sqrt{6.25} = 2.5$

b. $\mu = np = 80(.2) = 16$
 $\sigma^2 = np(1-p) = 80(.2)(.8) = 12.8$
 $\sigma = \sqrt{\sigma^2} = \sqrt{12.8} = 3.578$

c. $\mu = np = 100(.6) = 60$
 $\sigma^2 = np(1-p) = 100(.6)(.4) = 24$
 $\sigma = \sqrt{\sigma^2} = \sqrt{24} = 4.899$

d. $\mu = np = 70(.9) = 63$
 $\sigma^2 = np(1-p) = 70(.9)(.1) = 6.3$
 $\sigma = \sqrt{\sigma^2} = \sqrt{6.3} = 2.510$

e. $\mu = np = 60(.8) = 48$
 $\sigma^2 = np(1-p) = 60(.8)(.2) = 9.6$
 $\sigma = \sqrt{\sigma^2} = \sqrt{9.6} = 3.098$

f. $\mu = np = 1,000(.04) = 40$
 $\sigma^2 = np(1-p) = 1,000(.04)(.96) = 38.4$
 $\sigma = \sqrt{\sigma^2} = \sqrt{38.4} = 6.197$

4.57

a. For this experiment, there are $n = 200$ smokers (n identical trials). For each smoker, there are 2 possible outcomes: S = smoker enters treatment program and F = smoker does not enter treatment program. The probability of S (smoker enters treatment program) is the same from trial to trial. This probability is $P(S) = p = .05$. $P(F) = 1 - P(S) = 1 - .05 = .95$. The trials are independent and x = number of smokers entering treatment program in 200 trials. Thus, x is a binomial random variable.

b. $p = P(S) = .05$. Of all the smokers, only .05 or 5% enter treatment programs.

c. $E(x) = np = 200(.05) = 10$. For all samples of 200 smokers, the average number of

4.61

- a. Let x = number of beach trees damaged by fungi in 20 trials. Then x is a binomial random variable with $n = 20$ and $p = .25$.

$$P(x < 10) = P(x = 0) + P(x = 1) + \dots + P(x = 9)$$

$$\begin{aligned} &= \binom{20}{0} .25^0 .75^{20} + \binom{20}{1} .25^1 .75^{19} + \binom{20}{2} .25^2 .75^{18} + \dots + \binom{20}{9} .25^9 .75^{11} \\ &= .0032 + .0211 + .0669 + .1339 + .1897 + .2023 + .1686 + .1124 + .0609 + .0271 \\ &= .9861 \end{aligned}$$

- b. $P(x > 15) = P(x = 16) + P(x = 17) + \dots + P(x = 20)$

$$\begin{aligned} &= \binom{20}{16} .25^{16} .75^4 + \binom{20}{17} .25^{17} .75^3 + \binom{20}{18} .25^{18} .75^2 + \dots + \binom{20}{20} .25^{20} .75^0 \\ &= .000000356 + .000000027 + .000000001 + 0 + 0 = .000000384 \end{aligned}$$

4.63

- a. Let x = number of women out of 15 that have been abused. Then x is a binomial random variable with $n = 15$, S = woman has been abused, F = woman has not been abused, $P(S) = p = 1/3$ and $q = 1 - p = 1 - 1/3 = 2/3$.

$$\begin{aligned} P(x \geq 4) &= 1 - P(x < 4) = 1 - P(x = 0) - P(x = 1) - P(x = 2) - P(x = 3) \\ &= 1 - \frac{15!}{0!15!} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{15} - \frac{15!}{1!14!} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^{14} - \frac{15!}{2!13!} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^{13} \\ &\quad - \frac{15!}{3!12!} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^{12} \\ &= 1 - .0023 - .0171 - .0599 - .1299 = .7908 \end{aligned}$$

- b. For $p = .1$,

$$\begin{aligned} P(x \geq 4) &= 1 - P(x < 4) = 1 - P(x = 0) - P(x = 1) - P(x = 2) - P(x = 3) \\ &= 1 - \frac{15!}{0!15!} .1^0 .9^{15} - \frac{15!}{1!14!} .1^1 .9^{14} - \frac{15!}{2!13!} .1^2 .9^{13} - \frac{15!}{3!12!} .1^3 .9^{12} \\ &= 1 - .2059 - .3432 - .2669 - .1285 = .0555 \end{aligned}$$

- c. We sampled 15 women and actually found that 4 had been abused. If $p = 1/3$, the probability of observing 4 or more abused women is .7908. If $p = .1$, the probability of observing 4 or more abused women is only .0555. If $p = .1$, we would have seen a very unusual event because the probability is so small (.0555). If $p = 1/3$, we would have seen an event that was very common because the probability was very large (.7908). Since we normally do not see rare events, the true probability of abuse is probably close to $1/3$.

4.70

For the Poisson probability distribution

$$p(x) = \frac{10^x e^{-10}}{x!} \quad (x = 0, 1, 2, \dots)$$

the value of λ is 10.

4.72

- a. In order to graph the probability distribution, we need to know the probabilities for the possible values of x . Using Table III of Appendix A, with $\lambda = 10$:

$$p(0) = .000$$

$$p(1) = P(x \leq 1) - P(x \leq 0) = .000 - .000 = .000$$

$$p(2) = P(x \leq 2) - P(x \leq 1) = .003 - .000 = .003$$

$$p(3) = P(x \leq 3) - P(x \leq 2) = .010 - .003 = .007$$

$$p(4) = P(x \leq 4) - P(x \leq 3) = .029 - .010 = .019$$

$$p(5) = P(x \leq 5) - P(x \leq 4) = .067 - .029 = .038$$

$$p(6) = P(x \leq 6) - P(x \leq 5) = .130 - .067 = .063$$

$$p(7) = P(x \leq 7) - P(x \leq 6) = .220 - .130 = .090$$

$$p(8) = P(x \leq 8) - P(x \leq 7) = .333 - .220 = .113$$

$$p(9) = P(x \leq 9) - P(x \leq 8) = .458 - .333 = .125$$

$$p(10) = P(x \leq 10) - P(x \leq 9) = .583 - .458 = .125$$

$$p(11) = P(x \leq 11) - P(x \leq 10) = .697 - .583 = .114$$

$$p(12) = P(x \leq 12) - P(x \leq 11) = .792 - .697 = .095$$

$$p(13) = P(x \leq 13) - P(x \leq 12) = .864 - .792 = .072$$

$$p(14) = P(x \leq 14) - P(x \leq 13) = .917 - .864 = .053$$

$$p(15) = P(x \leq 15) - P(x \leq 14) = .951 - .917 = .034$$

$$p(16) = P(x \leq 16) - P(x \leq 15) = .973 - .951 = .022$$

$$p(17) = P(x \leq 17) - P(x \leq 16) = .986 - .973 = .013$$

$$p(18) = P(x \leq 18) - P(x \leq 17) = .993 - .986 = .007$$

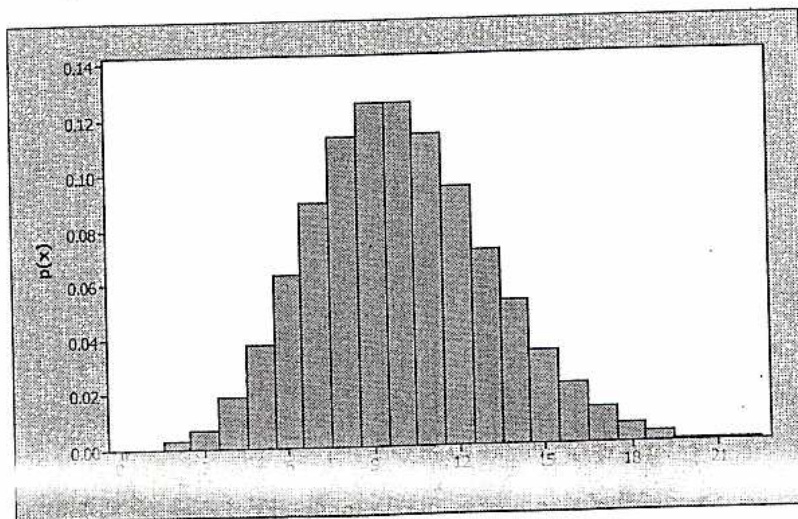
$$p(19) = P(x \leq 19) - P(x \leq 18) = .997 - .993 = .004$$

$$p(20) = P(x \leq 20) - P(x \leq 19) = .998 - .997 = .001$$

$$p(21) = P(x \leq 21) - P(x \leq 20) = .999 - .998 = .001$$

$$p(22) = P(x \leq 22) - P(x \leq 21) = 1.000 - .999 = .001$$

Using MINITAB, the probability distribution of x in graphical form is:



b. $\mu = \lambda = 10, \sigma^2 = \lambda = 10, \sigma = \sqrt{\lambda} = \sqrt{10} = 3.162$

4.74 a. For $\lambda = 1$, $P(x \leq 2) = .920$ (from Table III, Appendix A)

b. For $\lambda = 2$, $P(x \leq 2) = .677$

c. For $\lambda = 3$, $P(x \leq 2) = .423$

d. The probability decreases as λ increases. This is reasonable because λ is equal to the mean. As the mean increases, the probability that x is less than a particular value will decrease.

4.80

Let x = number of extrasolar planet transits for 10,000 stars. Then x has a Poisson distribution with $\lambda = 5.6$.

$$P(x > 10) = 1 - P(x \leq 10) = 1 - .972 = .028$$

4.81

a. $P(x = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = \frac{3.8^0 e^{-3.8}}{0!} = .0224$

b. $P(x = 1) = \frac{\lambda^1 e^{-\lambda}}{1!} = \frac{3.8^1 e^{-3.8}}{1!} = .0850$

c. $E(x) = \mu = \lambda = 3.8$

$$\sigma^2 = \lambda = 3.8$$

$$\sigma = \sqrt{3.8} = 1.9494$$

4.82

a. $\sigma^2 = \lambda = .03$

b. The characteristics of x , the number of casualties experienced by a deep-draft U.S. Flag vessel over a three-year period include:

1. The experiment consists of counting the number of deaths during a three-year time period.
2. The probability that a death occurs in a three-year period is the same for each three-year period.
3. The number of deaths that occur in a three-year period is independent of the number of deaths for any other three-year period.

Thus, x is a Poisson random variable.

c. $P(x = 0) = \frac{\lambda^0 e^{-\lambda}}{0!} = \frac{.03^0 e^{-.03}}{0!} = .9704$

4.92

For $N = 8$, $n = 3$, and $r = 5$,

(10)

$$a. \quad P(x=1) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{5}{1} \binom{8-5}{3-1}}{\binom{8}{3}} = \frac{\frac{5!}{1!4!} \frac{3!}{2!1!}}{\frac{8!}{3!5!}} = \frac{5(3)}{56} = .268$$

$$b. \quad P(x=0) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{5}{0} \binom{8-5}{3-0}}{\binom{8}{3}} = \frac{\frac{5!}{0!5!} \frac{3!}{3!0!}}{\frac{8!}{3!5!}} = \frac{1(1)}{56} = .018$$

$$c. \quad P(x=3) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{5}{3} \binom{8-5}{3-3}}{\binom{8}{3}} = \frac{\frac{5!}{3!2!} \frac{3!}{0!3!}}{\frac{8!}{3!5!}} = \frac{10(1)}{56} = .179$$

$$d. \quad P(x \geq 4) = P(x=4) + P(x=5) = 0$$

Since the sample size is only 3, there is no way to get 4 or more successes in only 3 trials.

4.94

With $N = 10$, $n = 5$, and $r = 7$, x can take on values 2, 3, 4, or 5.

$$a. \quad P(x=2) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{7}{2} \binom{10-7}{5-2}}{\binom{10}{5}} = \frac{\frac{7!}{2!5!} \frac{3!}{3!0!}}{\frac{10!}{5!5!}} = \frac{21(1)}{252} = .083$$

$$P(x=3) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{7}{3} \binom{10-7}{5-3}}{\binom{10}{5}} = \frac{\frac{7!}{3!4!} \frac{3!}{2!1!}}{\frac{10!}{5!5!}} = \frac{35(3)}{252} = .417$$

$$P(x=4) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{7}{4} \binom{10-7}{5-4}}{\binom{10}{5}} = \frac{\frac{7!}{4!3!} \frac{3!}{1!2!}}{\frac{10!}{5!5!}} = \frac{35(3)}{252} = .417$$

$$P(x=5) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{7}{5} \binom{10-7}{5-5}}{\binom{10}{5}} = \frac{\frac{7!}{5!2!} \frac{3!}{0!3!}}{\frac{10!}{5!5!}} = \frac{21(1)}{252} = .083$$

The probability distribution of x is tabulated below:

x	$p(x)$
2	.083
3	.417
4	.417
5	.083

(continued)

4.94 (continued)

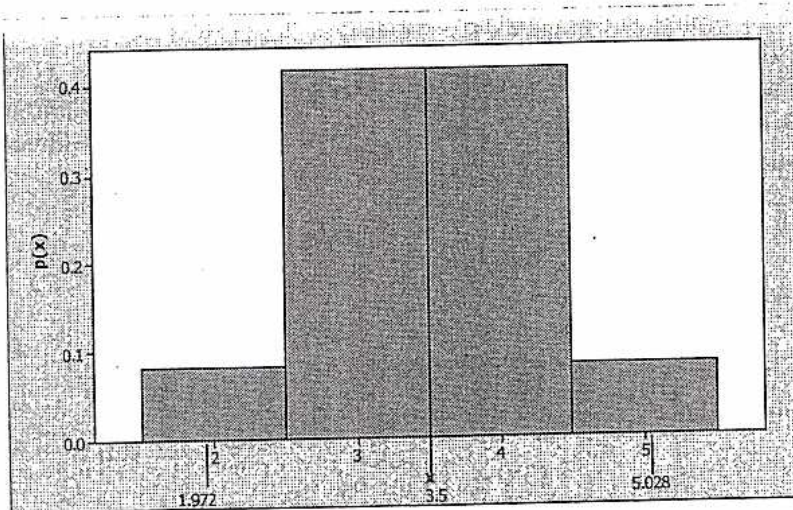
b. $\mu = \frac{nr}{N} = \frac{5(7)}{10} = 3.5$

$$\sigma^2 = \frac{r(N-r)n(N-n)}{N^2(N-1)} = \frac{7(10-7)5(10-5)}{10^2(10-1)} = \frac{525}{900} = .5833$$

$$\sigma = \sqrt{.5833} = .764$$

c. $\mu \pm 2\sigma \Rightarrow 3.5 \pm 2(.764) \Rightarrow 3.5 \pm 1.528 \Rightarrow (1.972, 5.028)$

The graph of the distribution is:



d. $P(1.972 < x < 5.028) = P(2 \leq x \leq 5) = 1.000$

4.98

a. Let x = number of facilities chosen that treat hazardous waste on-site in 10 trials. For this problem, $N = 209$, $r = 8$, and $n = 10$.

$$E(x) = \mu = \frac{nr}{N} = \frac{10(8)}{209} = .383$$

$$\frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{8}{1} \binom{201}{9}}{\binom{209}{10}} = \frac{8! 201!}{10! 199!} = .0002$$

- 4.100 a. Let x = number of defective items in a sample of size 4. For this problem, x is a hypergeometric random variable with $N = 10$, $n = 4$, and $r = 1$. You will accept the lot if you observe no defectives.

$$P(x=0) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{1}{0} \binom{10-1}{4-0}}{\binom{10}{4}} = \frac{\frac{1!}{0!1!} \frac{9!}{4!5!}}{\frac{10!}{4!6!}} = \frac{1(84)}{210} = .4$$

- b. If $r = 2$,

$$P(x=0) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{2}{0} \binom{10-2}{4-0}}{\binom{10}{4}} = \frac{\frac{2!}{0!2!} \frac{8!}{4!4!}}{\frac{10!}{4!6!}} = \frac{1(70)}{210} = .333$$

- 4.102 Let x = number of British bird species sampled that inhabit a butterfly hotspot in 4 trials. Because the sampling is done without replacement, x is a hypergeometric random variable with $N = 10$, $n = 4$, and $r = 7$.

a.
$$P(x=2) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{7}{2} \binom{10-7}{4-2}}{\binom{10}{4}} = \frac{\frac{7!}{2!5!} \frac{3!}{2!1!}}{\frac{10!}{4!6!}} = \frac{21(3)}{210} = .3$$

- b. $P(x \geq 1) = 1$ because the only values x can take on are 1, 2, 3, or 4.