UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

STAT 213 L05 MIDTERM EXAM Nov. 1, 2005 TIME:60 minutes

(Put your ID number at the top of page 2) ~~NAME:~~ Solutions / Marking Key

Marks

10 1. Six participants in a bike race had the following finishing times in minutes:

18, 12, 16, 23, 11, 13 .

For this sample of finishing times, compute the following statistics:

2 (a) the mean;

$$\bar{X} = \frac{\Sigma X_i}{n} = \frac{18+12+16+23+11+13}{6} = \frac{93}{6} = \underline{\underline{15.5}} \text{ min}$$

2 (b) the median; ordered observations are: 11, 12, 13, 16, 18, 23

$$\text{med} = \frac{13+16}{2} = \underline{\underline{14.5}} \text{ min.}$$

2 (c) the interquartile range;

$6(.25) = 1.5$, round up to 2 : 2^{nd} ordered obs = 12 } ← 1
$6(.75) = 4.5$, round up to 5 : 5^{th} ordered obs = 18 }

$18 - 12 = \underline{\underline{6}}$ min. ← 1

1 (d) the 70th percentile;

$6(.7) = 4.2$, round up to 5, so 70^{th} %-ile = 5^{th} ordered obs. = $\underline{\underline{18}}$ min.

3 (e) the standard deviation.

$$S^2 = \frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1} = \frac{\overbrace{1543 - \frac{(93)^2}{6}}^{1}}{5} = \underline{20.3} \text{ min}^2 \leftarrow 1$$

so $S = \sqrt{20.3} = \underline{\underline{4.506}}$ min ← 1

ID number: ______________

Marks

10 2. In an experiment to study the relationship between the dose of a stimulant and the time a stimulated subject takes to respond to an auditory signal, the following data were recorded:

Dose (x) (milligrams)	1	3	4	8	9
Reaction time (y) (seconds)	1	2	2	3	4

7 (a) Calculate the correlation coefficient.

x_i	y_i	x_i^2	$x_i y_i$	y_i^2
1	1	1	1	1
3	2	9	6	4
4	2	16	8	4
8	3	64	24	9
9	4	81	36	16
25	12	171	75	34

$$SS_{xy} = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n} = 75 - \frac{(25)(12)}{5} = 15 \quad (2)$$

$$SS_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = 171 - \frac{(25)^2}{5} = 46 \quad (2)$$

$$SS_{yy} = \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 34 - \frac{(12)^2}{5} = 5.2 \quad (1)$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}\,SS_{yy}}} = \frac{15}{\sqrt{(46)(5.2)}} = 0.9699 \quad (1)$$

3 (b) Find the equation of the best-fitting line, with x as the independent variable and y as the dependent variable.

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{15}{46} = 0.3261 \leftarrow (1)$$

$$b_0 = \bar{y} - b_1\bar{x} = \frac{12}{5} - 0.3261\,\frac{25}{5} = 0.7695 \leftarrow (1)$$

So equation is: $\hat{y} = 0.7695 + 0.3261x \leftarrow (1)$

Marks

10 3. A jar contains four green marbles and three red marbles. Three of the seven marbles are selected at random, without replacement.

2 (a) Find the probability that exactly two of the selected marbles are green.

$\begin{bmatrix} 4G \\ 3R \end{bmatrix}$

$$P[2 \text{ Green}] = P(GGR) + P(GRG) + P(RGG)$$

$$= \frac{4}{7}\frac{3}{6}\frac{3}{5} + \frac{4}{7}\frac{3}{6}\frac{3}{5} + \frac{3}{7}\frac{4}{6}\frac{3}{5} = 3\,\frac{4}{7}\frac{3}{6}\frac{3}{5} = \frac{18}{35}$$

(1 mark for the wrong answer 6/35). $(= 0.5143)$

3 (b) Find the probability that exactly two of the selected marbles are red.

$$P(RRG) + P(RGR) + P(GRR) = 3\,\frac{3}{7}\frac{2}{6}\frac{4}{5} = \frac{12}{35} = 0.3429$$

(1 mark for the wrong answer 4/35)

2 (c) Find the probability that the number of green marbles selected is at least one.

$$P[\text{at least one } G] = 1 - P(RRR) = 1 - \frac{3}{7}\frac{2}{6}\frac{1}{5} = \frac{34}{35} = 0.9714$$

3 (d) Find the conditional probability that exactly two of the selected marbles are green, given that the number of green marbles selected is at least one.

$$P[2G \mid \geq 1\,G] = \frac{P[2G \cap (\geq 1\,G)]}{P[\geq 1\,G]} = \frac{P(2G)}{P(\geq 1\,G)}$$

$$= \frac{18/35}{34/35} = \frac{18}{34} = \frac{9}{17} = 0.5294$$

10 4. Consider the following game:

Two fair dice are rolled. If the sum of the two numbers that come up is 4 or less, you win \$1. If the sum is 9 or more, you win \$3. Otherwise (that is, if the sum is 5, 6, 7 or 8), you win \$2.

Let W denote the amount that you win when you play this game.

8 (a) Obtain the probability distribution of W.

w	P(W=w)
1	6/36 (=1/6) ← 2
2	20/36 (=5/9) ← 2
3	10/36 (=5/18) ← 2

2 marks for get values of W correct

$P(W=1) = P[\text{sum} \leq 4]$

$= P[(1,1),(1,2),(1,3),(2,1),(2,2),(3,1)]$

$= 6/36$

$P[W=3] = P[(3,6),(4,5),(4,6),(5,4),(5,5),(5,6),(6,3),(6,4),(6,5),(6,6)]$

$= 10/36$

$P[W=2] = 1 - P(W=1) - P(W=3)$

$= 1 - \frac{6}{36} - \frac{10}{36} = \frac{20}{36}$

2 (b) Find the expected value and variance of W.

$EW = \sum w\,p(w) = 1\cdot\frac{3}{18} + 2\cdot\frac{10}{18} + 3\cdot\frac{5}{18} = \frac{38}{18} = \frac{19}{9} = \$\,2.111$ ← 1

$E(W^2) = 1\cdot\frac{3}{18} + 4\cdot\frac{10}{18} + 9\cdot\frac{5}{18} = \frac{88}{18} = \frac{44}{9}$

$\Rightarrow Var(W) = E(W^2) - (EW)^2 = \frac{44}{9} - \left(\frac{19}{9}\right)^2 = \frac{396-361}{81}$

$= \frac{35}{81} = 0.4321\ (\$)^2$ ← 1