

STAT 213
Assignment #4

MINITAB INSTRUCTIONS

CALC ⇒ Probability Distributions ⇒ normal.

Finding the area above and below a Z-value under the Standard Normal Curve

For a given Z-value, you are required to find a probability. In the dialog box, which corresponds to the Normal distribution, you have three choices:

- Probability**
- Cumulative probability**
- Inverse cumulative probability**

Click **Cumulative probability**. This will calculate the cumulative probability associated with a specific Z-value (or the area under the Standard Normal Curve to the **left** of a specific Z-value.)

The middle of the dialog box has 2 options:

- Mean**
- Standard Deviation**

The default values for the **Mean** and **Standard Deviation** are **0** and **1**, respectively. There is no need to change these values, so just leave these as is.

In the bottom portion of the dialog box, select **Input constant**. It is in this box that you enter a specific Z-value. Once you have completed this, either press return or “click” on **OK**. In the upper portion of your screen, or the command module, MINITAB will return the area to the **left** of the Z-value you have entered. Note that when this routine is employed, the probability returned is **ALWAYS THE AREA TO THE LEFT OF** the Z-value entered above or $P(Z < z)$

For practice, try question 1 using this routine.

1. Given that Z is a standard normal random variable, compute the following probabilities:

- (a) $P(-0.72 \leq Z \leq 0)$ (.2642)
- (b) $P(-0.35 \leq Z \leq 0.35)$ (.2736)
- (c) $P(0.22 \leq Z \leq 1.87)$ (.3822)
- (d) $P(Z \leq -1.02)$ (.1539)
- (e) $P(Z \geq -0.88)$ (.8106)
- (f) $P(Z \geq 1.38)$ (.0838)
- (g) $P(-0.34 \leq Z \leq 2.33)$ (.6232)

Finding a Z-value for a given area (or probability under the Standard Normal Curve)

For a given probability, you are required to find a Z-value that corresponds to this probability. This requires the use of the **Inverse cumulative probability** routine in the dialog box employed above.

“Click” on the circle which corresponds to **Inverse cumulative probability** and just as was done previously, do not touch the box labeled **Mean** and **Standard Deviation**.

This routine needs an area, and will subsequently find the Z-value which matches up with the area entered. Just as was done above, move your mouse down to the bottom portion of the dialog box and “click” on the circle which corresponds to **Input constant**. Previously you entered a Z-value here. But now you want to find a Z-value for a given area, or probability. So the number you will enter in the **Input constant** box is a probability, or an area to the left of the Z-value in question. Once you have entered the correct probability,

either press return or “click” on **OK**. MINITAB will return a Z-value in the command module on the upper portion of your screen.

A good rule-of-thumb in these types of problems is to draw your standard normal curve and piece together the areas given. The Z-value will be given when you specify the area to the left of that value. Practice this routine on question 2.

2. Given that Z is a standard normal random variable, determine Z_o if it is known that:

- (a) $P(-Z_o \leq Z \leq Z_o) = 0.90$ (1.645)
- (b) $P(-Z_o \leq Z \leq Z_o) = 0.10$ (.1257)
- (c) $P(Z \geq Z_o) = 0.20$ (.842)
- (d) $P(-1.66 \leq Z \leq Z_o) = 0.25$ (-.529)
- (e) $P(Z \leq Z_o) = 0.40$ (-.253)
- (f) $P(Z_o \leq Z \leq 1.80) = 0.20$ (.720)

3. Every year around Halloween, many street signs are defaced. The average repair cost per sign is \$68.00 and the standard deviation is \$12.40.

- (a) What is the probability that a defaced sign’s repair cost will exceed \$85.00? (.0852)
- (b) In one particular year, 36 signs were vandalized. What is the probability that the average repair cost of these signs is between \$67.00 and \$70.00? (0.5184)

4. According to a article in the January 1991 issue of *Health* magazine, the cost of a root canal usually ranges from \$200 to \$700. Suppose the mean cost for root-canal therapy is \$450 and the standard deviation is \$125. If a sample of 100 dentists from across the country was taken, what is the probability that the mean cost of a root-canal will fall between \$425 and \$475 (.9554)

5. The time a recreational skier takes to go down a downhill course has a normal distribution with a mean of 12.3 minutes and a standard deviation of 0.4 minutes.

- (a) What is the probability that the skier will take between 12.1 and 12.6 minutes to complete a run on the course? (.4649)
- (b) What is the maximum time (in minutes) the skier must have for the time to be classified “among the fastest 10% of his times”? (11.7872)
- (c) The times for a random sample of 4 of the skier’s runs is considered. What is the probability that the average time for this sample is more than 12.75 minutes? (0.0122)

6. The time it takes Bob to get from home to his office follows a normal distribution. The probability that it takes him less than 3 minutes is 0.345. The probability that it takes him more than 10 minutes is 0.01. Find the average time and variance (μ and σ^2) of this normal distribution. (4.0256, 6.5746)

7. “Your Eyes”, a daily eye ware retail store, serves an average of 14.3 customer per day. Assume that the distribution of the number of customer served per day has a standard deviation of 5.9. What is the probability that the average number of customers served per day will be:

- (a) at least 15, based on a random sample of 50 days? (.2008)
- (b) less than 14, based on a random sample of 50 days? (0.3596)

8. A random sample of size 64 is taken from a normal population with $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of the sample will

- (a) exceed 52.9? (0.0388)
- (b) fall between 50.5 and 52.3? (0.7104)
- (c) be less than 50.6? (0.1733)

9. Electrical connectors last on average 18.2 months with a standard deviation of 1.7 months. Assume that the life of the connectors is normally distributed.
- The manufacturer agrees to replace, free of charge, any connectors that fail within 17 months of installation. What percentage of the connectors can he expect to have replaced free of charge? (.2404)
 - The manufacturer does not want to have to replace more than 2.5% of the connectors free of charge. What should he set the life at for free replacement? (14.868)
 - What is the probability that the **total** lifetime of 24 connectors will exceed 38 **years**? (0.0105)
10. The lengths of pregnancies are normally distributed with a mean of 268 days and a standard deviation of 15 days.
- If one pregnant female is randomly selected, find the probability that her length of pregnancy is less than 260 days. (.2981)
 - If 25 randomly selected females are put on a special diet just before they become pregnant, find the probability that their lengths of pregnancy have a mean that is less than 260 days (assuming that the diet has no effect). (.0038)
 - If the 25 females do have a mean of less than 260 days, should the medical supervisors be concerned?

Normal approximation to Binomial.

Note: answers may vary slightly due to rounding

- A newspaper advertisement claims that 55% of the people who wear contact lenses experience no difficulty. In a random sample of 300 people who have purchased contact lens,
 - What's the probability of at least 150 people having no problems? {.9641}
 - What's the probability of having between 145 and 170 (inclusive) people having no problems? {.7302}
 - What's the probability of having less than 160 having no problems? {.2611}
- According to one study, $\frac{2}{3}$ of all Canadians have at least 2 televisions. In a random sample of 1000 Canadians,
 - What's the probability of exactly 668 Canadians having at least 2 televisions? (use approximation) {.0239}
 - What's the probability of between 640 and 670 (exclusively) Canadians having at least 2 televisions? {.5361}
 - What's the probability of greater than 670 Canadians having at least 2 televisions? {.3974}
- Estimate the probability of getting at least 52 girls in 100 births. Assume that boys and girls are equally likely. [.3821]
- Estimate the probability of passing a true/false test of 50 question if 60% (or 30 correct answers) is the minimum passing grade and all responses are random guesses. [.1020]