

**Exercise 10: Statistics 213 (L05) - Fall 2007**

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1. The strength of a pesticide dosage is often measured by the proportion of pests the dose will kill. A particular dosage of rat poison is fed to 250 rats. Of these rats, 215 died due to the poison.
  - (a) Test whether it could be concluded, at the 0.10 level of significance, that the true proportion of all rats that would succumb to the dosage is more than 0.85.
  - (b) Find a 95% confidence interval estimate for the proportion of all rats that would succumb to the dosage.
2. A hypnotist claims to have at least a 97% success rate for curing smoking addiction. A random sample of 180 of the hypnotist's patients attempting to quit smoking contained 170 patients who had quit smoking.
  - (a) Is there sufficient evidence to reject the hypnotist's claim. Use  $\alpha = 0.05$ .
  - (b) Find the  $p$ -value for the test.
3. The Glen Valley Steel Company manufactures steel bars. The production process should turn out steel bars with an average length of at least 2.8 ft. when the process is working properly (the lengths of the steel bars are normally distributed). A sample of 25 bars is selected from the production line. The sample gives an average length of 2.43 ft. and a standard deviation of 0.20 ft. The company wishes to determine whether the machine needs adjusted. Carry out an approximate hypothesis test at the 5% level of significance.
4. Scores on an aptitude test used for determining admission to graduate study in business are known to be normally distributed with a mean of 500 and a population standard deviation of 100. If a random sample of 12 applicants from Stephen College have a sample mean of 537
  - (a) Is there any evidence at the 1% level of significance that their mean score is different from the mean expected of all applicant ?
  - (b) Find the  $p$ -value for the above test.
5. Past experience indicates that the average monthly long-distance telephone bill per household is \$10.12. After an advertising campaign that encouraged people to make long-distance telephone calls more frequently, a random sample of 57 households revealed that the mean monthly long-distance bill was \$10.98 with standard deviation of \$3.27. Can we conclude at the 10% level of significance that the advertising campaign was successful?
6. To access a new method for measuring phosphorus levels in soils, a sample of 5 soil specimens, each with a true phosphorus content of 548 mg/kg, is analyzed using the method. The sample mean and standard deviation were 557 mg/kg and 10 mg/kg respectively. Of interest is whether the mean phosphorus level reported by the new method differs from 548 mg/kg. Assume that the phosphorus measurements are normally distributed. Carry out an appropriate hypothesis test at  $\alpha = 0.05$ .

1. Let  $Y$  be the number of rats that succumb to poison  $\sim \text{Binomial}(250, p)$ . (a).  $H_0 : p = 0.85$  vs  $H_a : p > 0.85$ ,  $\hat{p} = \frac{215}{250} = 0.86$ ,  $n = 250$ ,  $Z_{cal} = 0.44 < 1.28 = z_{0.90}$ . Do not reject  $H_0$ . There is no evidence at  $\alpha = 0.10$  that true population of rats that would succumb is more than 0.85.
2. Let  $Y$  be the number of patients out of 170 who had quit smoking  $\sim \text{Binomial}(180, p)$  (a).  $H_0 : p = 0.97$  vs  $H_a : p < 0.97$ , calculated test statistic,  $z = -2.02 < -1.645 = -z_{0.05}$ . Reject  $H_0$  and conclude that there is sufficient evidence at  $\alpha = 0.05$  to reject claim of at least a 97% success rate. (b).  $p\text{-value} = P(Z < -2.02) = 0.0217$
3.  $H_0 : \mu = 2.8$  vs  $H_a : \mu < 2.8$ , calculated test statistic,  $t_{24} = -9.25 < t_{24,0.05} = -1.711$ : the critical value, reject  $H_0$  and conclude at the 5% level of significance that average length of steel bars is less than 2.8ft, i.e. machine needs adjustment.
4. (a).  $H_0 : \mu = 500$  vs  $H_a : \mu \neq 500$ ,  $Z_{cal} = 1.28 < 2.575 = z_{0.995}$ , therefore there is no reason to reject  $H_0$ . i.e. there is no evidence at 0.01 level of significance that the mean score of Stephen College applicants differs from 500. (b).  $p\text{-value} = 2P(Z > 1.28) = 0.2006$ .
5.  $\mu = 10.12$ , but no information about  $\sigma^2$ . The sample size,  $n = 57$  is large enough for the CLT to apply ( $\bar{X}$  approximates to Normal).  $H_0 : \mu = 10.12$  vs  $H_a : \mu > 10.12$ ,  $Z_{cal} = 1.986 > z_{0.90} = 1.28$ , reject  $H_0$ , and conclude at the  $\alpha = 0.10$  level of significance that campaign was successful.
6.  $H_0 : \mu = 548$  vs  $H_a : \mu \neq 548$ , the calculated test statistic,  $t_4 = 2.0125 < t_{0.975,4} = 2.7764$ , do not reject  $H_0$  and conclude that there is insufficient evidence that the the mean differ from 548.