Exercise 3 and 4: Statistics 213 (L05) - Fall 2007

1. What, if anything, is wrong with the following statements ?
   a. the probability that a student will get an A in their STAT course is 0.16, and the probability that they will not get an A is 0.90.
   b. the probabilities that a student writing an exam will get an A, B, C, D, or F are respectively: $0.1, 0.3, 0.3, 0.1, 0.05$
   c. the probability that a student will get an A in their computer course is 0.16, and the probability that they will not get an A in their STAT is 0.9.
   d. the probability that a computer student will write an error free program is 0.15 and the probability that they will make no more than 1 error in the program is 0.12.

2. If A and B are mutually exclusive events, what can you say about:
   a. $P(A \cup B)$?
   b. $P(A \cap B)$?
   c. are A and B independent ?

3. You are given the following information on events, A, B and C.

   $P(A) = 0.4$, $P(B) = 0.2$, $P(C) = 0.1$, $P(A|B) = 0.3$ and $P(A \cap C) = 0.04$
   a. Find $P(A \cap B)$, $P(A \cup B)$, $P(A|C)$, and $P(C^c)$
   b. Are A and B mutually exclusive ? Explain.
   c. Are A and B independent ? Explain.
   d. Are A and C independent? Explain.

4. Consider an experiment where an experiment where a die is rolled twice.
   a. Find the sample space:
   b. Let A be the event that a sum of 4 or less is observed.
   c. Let B be the event that the first roll is a 2.
   d. Let C be the event that a sum of 6 is observed.
   e. Find A^c.
   f. Find $A \cup B$ and $A \cup C$.
   g. Find $A \cap B$ and $A \cap C$.
   h. Find the probabilities for the (a)-(g).

5. Each morning coffee is prepared for the entire office staff by one of 3 employees, depending on who first arrives at work. Veronica (V) arrives first 20% of the time; Gita (G) and Michael (M) are each first to arrive on half of the remaining mornings. The probability that the coffee is bitter (B) when it is prepared by Veronica is 0.1, while the corresponding probabilities when it is prepared by Gita and Michael are 0.2 and 0.3 respectively. If you arrive one morning and find that the coffee is bitter, what is the probability that it was prepared by Veronica ?

6. An aerospace company has submitted bids on two separate federal government defense contracts, A and B. The company feels that it has a 60% chance of winning contract A and 30% chance of winning contract B. If it wins contract B, it believes it has an 80% chance of winning contract A.

   a. What is the probability that the company will win both contracts ?
   b. Are the events of winning contract A and winning contract B independent ? Explain.
   c. What is the probability that the company will win at least one of the contracts ?
   d. What is the probability that the company will win neither contract ?
   e. What is the probability that the company will win contract A but not contract B ?
   f. If the company wins contracts B, what is the probability that it will not win contract A ?
   g. If the company wins contracts A, what is the probability that it will win contract B ?
   h. If the company doesn't win contract A, what is the probability that it will win contract B ?
   i. Are the events of winning contract A and winning contract B mutually exclusive ? Explain.

7. Suppose that the aerospace company in the previous example feels that it has a 50% chance of winning contract A and a 40% chance of winning contract B. It also believes that winning contract A is independent of winning contract B.

   a. What is the probability that the company will win both contracts ?
   b. What is the probability that the company will win at least one of the contracts ?

8. Each week a retail outlet accepts delivery of a certain item from 3 different suppliers, A, B and C. All the items received from the three suppliers are put into an empty bin. Supplier A provides 50% of these items, while B and C each supply 25%. From past experience, it is known that 2% of the items supplied by A are defective, 2% of the items supplied by B are defective, while 4% of the items supplied by C are defective. Suppose that an item is selected at random from the bin.

   a. What is the probability that it is defective ?
   b. If the item is found to be defective, what is the probability that it came from supplier C ?

9. In a study of consumer planning for the purchase of a new car, 1000 individuals were asked whether they were planning to buy a new car in the next 12 months. A year later the same people were interviewed again to find out whether they had bought a new car or not. The responses are given in the table below.

	buyer (B)	non-buyer (B^c)	totals
planned to buy (P)	200	50	250
didn't plan to buy (P^c)	100	650	750
totals	300	700	1000

   If an individual is selected at random from among those interviewed, find their probabilities:

   a. has bought a new car

b. planned to buy a new car
c. did not plan to buy a new car
d. has not bought a new car
e. planned to buy and actually bought a new car
f. planned to buy but did not buy a new car
g. did not plan to buy and actually did not buy a new car
h. planned to buy a new car or actually bought a new car
i. planned to buy a new car or did not plan to buy a new car
j. actually bought a new car if they planned to buy one
k. did not buy a new car if they had not planned to buy one
l. are the events B and P independent ?

10. From a bag containing 5 white, 2 blacks, and 13 red balls. 2 balls are drawn at random without replacement.

    a. both will be white
    b. neither will be white
    c. the two balls will be of the same color

Text book: chapter 14 and 15: odd numbers

Hints

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1. (a). wrong (b). wrong (c). nothing wrong (d). wrong

2. A and B: mutually exclusive $\Longrightarrow P(A \cap B) = 0$ (a). $P(A \cup B) = P(A) + P(B)$, (c). $P(B|A) \neq P(B)$

3. (a). $P(A \cap B) = P(B)P(A|B) = 0.06$, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.54$, $P(A|C) = \frac{P(A \cap C)}{P(C)} = 0.4$, $P(C^c) = 1 - P(C) = 0.9$ (b). no (c). no (d). yes

5. Bayes' Rule: $P(V) = 0.2$, $P(G) = 0.4$, $P(M) = 0.4$, $P(B|V) = 0.1$, $P(B|G) = 0.2$, $P(B|M) = 0.3$, $P(V|B) = \frac{P(V \cap B)}{P(B)} = \frac{0.2 \times 0.1}{0.22} = 0.09$

6. $P(A) = 0.6, P(B) = 0.3, P(A|B) = 0.8$ (a). $P(A \cap B) = 0.24$, (b). $P(A|B) \neq P(A)$, (c). $P(A \cup B) = 0.66$, (d). $P(A^c \cap B^c) = 0.39$, (e). $P(A \cap B^c) = 0.36$, (f). $P(A^c|B) = 0.2$, (g). $P(B|A) = 0.4$, (h). $P(B|A^c) = 0.15$, (i). $P(A \cap B) \neq 0$.

8. Bayes' Rule: $P(A) = 0.5$, $P(B) = 0.25$, $P(C) = 0.25$, $P(D|A) = 0.02$, $P(D|B) = 0.02$, $P(D|C) = 0.04$, $P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D) = 0.025$, $P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{0.01}{0.025} = 0.4$

9. (a). $P(B) = 0.3$, (b). $P(P) = 0.25$, (c). $P(P^c) = 0.75$, (d). $P(B^c) = 0.7$, (e). $P(P \cap B) = 0.2$, (f). $P(P \cap B^c) = 0.05$, (g). $P(P^c \cap B^c) = 0.65$, (h). $P(P \cup B) = 0.35$, (i). $P(P \cup P^c) = 1$, (j). $P(B|P) = 0.8$, (k). $P(B^c|P^c) = 0.86$, (l). $P(B|P) \neq P(B)$

10. (a). $P(W \cap W) = P(W)P(W|W) = \frac{5}{20}\frac{4}{19}$, (b). $P(W^c \cap W^c) = P(W^c)P(W^c|W^c) = \frac{15}{20}\frac{14}{19}$, (c). $P((W \cap W) \cup (B \cap B) \cup (R \cap R)) = P(W \cap W) + P(B \cap B) + P(R \cap R) = \frac{1}{19} + \frac{2}{20}\frac{1}{19} + \frac{13}{20}\frac{12}{19}$