

Exercise 5: Statistics 213 (L05) - Fall 2007

1. Let X be a random variable with the probability distribution given in the following table: Find

X	0	1	2	3	4	5
$P(X)$	0.05	0.10	0.200	0.40	0.20	0.05

$E(X)$ and $V(X)$.

2. Choose a Canadian household at random and let the random variable X be the number of persons living in the household. The probability distribution of X is following:

outlook	1	2	3	4	5	6	7
probability	0.237	0.317	0.178	0.157	0.070	0.026	0.015

- a. What is $P(X \geq 5)$?
 - b. What is $P(X > 5)$?
 - c. What is $P(2 < X \leq 4)$?
 - d. What is $P(X \neq 1)$?
 - e. What is the probability of the event that a randomly chosen household contains more than two persons in terms of the random variable X ?
3. Consider a multiple choice exam in which each question has four possible answers. if a person guesses the answer to each question, then the probability of a correct answer is 0.25, and the probability of a wrong answer is 0.75. Suppose the test consists of three different questions.
- a. Draw a tree diagram for the possible outcomes.
 - b. Define a random variable X to represent the number of correct answers that an individual gets when writing the test, find the possible X values.
 - c. Find the probability distribution of a random variable, X .
 - d. What is the probability that at least one question is answered correctly ?
 - e. What is the probability that at most two questions are answered correctly ?
 - f. Find the mean and standard deviation of X .
4. Suppose we had 100 individuals write the test in the question (1) in such a way where they each guessed the answer to every question.

number of correct answers	0	1	2	3
number of tests	37	44	15	4

- a. For this sample of 100 tests, find the mean number of correct answer per test.
 - b. For this sample of 100 tests, find the sample standard deviation of correct answer per test.
5. In a weekly lottery, there is 1 prize for \$10,000, 2 prizes of \$ 5000, 500 prizes of \$100, and 10,000 prizes of \$3. Suppose that 100,000 tickets are sold, each costing \$ 3. If you purchase a ticket, what are your expected winnings (gain) ?

6. Let X , Y and Z three independent random variables such that $E(X) = 9$, $E(Y) = 4$, $E(Z) = 12$, $V(X) = 10$, $V(Y) = 12$, $V(Z) = 20$. Find $E(3X + 2 - 4Z)$, $V(10X - 4)$, $V(2Y - 4Z)$, $E(X - Y - Z)$, $V(X + 2Y + 3Z)$ and $V(5Z)$.
7. A book salesperson estimates the probabilities for the number of books sold in a week as follow:

number of sold	0	1	2	3	4	5
probability	0.05	0.10	0.40	0.20	0.15	0.10

assume that the number of sales in one week is independent of the number in any other:

- What is the probability, for a given week, that more than 3 books will be sold ?
- Find the expected number of books sold in a week.
- Find the standard deviation of the number of book sold in a week.
- The salesperson receives a weekly salary of \$250, plus a commission of \$50 for each book sold. Find the expected value of the total amount the salesperson will receive each week.
- Find the standard deviation of the total weekly amount.
- What is the probability that the sales person will sell exactly 4 books in each week of a 3 week period ?
- What is the probability that the salesperson will sell at least 4 books in each week of a 3 week period ?

Solutions

- $E(X) = 2.75$, $V(X) = 1.3875$
- (a). 0.111 (b). 0.041 (c). 0.335 (d). 0.763 (e). $P(X > 2) = 0.446$
- (b). $X = 0, 1, 2$ and 3 , (c). $P(X = 0) = 0.4219$, $P(X = 1) = 0.4219$, $P(X = 2) = 0.1406$, $P(X = 3) = 0.0156$, (d). $P(X \geq 1) = 0.5781$, (e). $P(X \leq 2) = 0.9844$, (f). $E(X) = 0.75$, $V(X) = \sigma^2 = 0.5625$, $\sigma = 0.75$
- (a). $\bar{x} = \frac{0 \times 37 + 1 \times 44 + 2 \times 15 + 3 \times 4}{100} = 0.86$, (b). $\hat{\sigma}^2 = \frac{(0-0.86)^2 \times 37 + (1-0.86)^2 \times 44 + (2-0.86)^2 \times 15 + (3-0.86)^2 \times 4}{100-1} = 0.667$, $\hat{\sigma} = 0.8167$
- Let X : *gain*, $X = 10000 - 3 = 9997$, $5000 - 3 = 4997$, $100 - 3 = 97$, $3 - 3 = 0$, $0 - 3 = -3$, $P(X = 9997) = \frac{1}{100000}$, $P(X = 4997) = \frac{2}{100000}$, $P(X = 97) = \frac{500}{100000}$, $P(X = 0) = \frac{10000}{100000}$, $P(X = -3) = \frac{89497}{100000}$, (a). expected winning ≈ -2 .
- 13, 1000, 368, -7, 238, 500
- Let X : the number of books sold (a). $P(X > 3) = 0.25$, (b). $E(X) = 2.6$, $V(X) = \sigma_X^2 = 1.64$, $\sigma_X = 1.28$, (d). Let T : the total amount received per a week, $T = 250 + 50X$, $E(T) = \$380$ (e). $\sigma_T = \$64$ (f). Let F be the event that person will sell exactly 4 book in a week, $P(F) = 0.15$, Want $P(F \cap F \cap F) = 0.003375$, (g). Let A be the event that person will sell at least 4 books in a week, $P(A) = 0.25$, Want $P(A \cap A \cap A) = 0.0156$