Exercise 6: Statistics 213 (L05) - Fall 2007

1. In each of following problems decide whether the appropriate model is Binomial, Hypergeometric, Poisson. Give the value of parameters of distribution you choose (i.e. (n, p) for Binomial, $(n, M, N - M)$ for Hypergeometric; λ for Poisson).

   a. Computation of the probability that at least 20 people respond to the mailing of 100 advertising circulars when it is known that the usual response is 15%

   b. Determination of the probability that a manufacturer receives no defective machines in a shipment of 3 machines from a firm that has 3 defective and 7 non-defective machine in its warehouse.

   c. Computation of the probability that there are more than 5 arrivals to a hospital emergency room during a given 24-hour day when, on average, arrivals occur randomly and independently at the rate of 1 every 6 hours

   d. 10 mill workers are chosen at random from a lumber manufacturing plant in which 70 workers are union members and 38 are not. Management wishes to determine the probability that at least 5 non-union members are chosen in the sample

   e. Competitive bids are required for the purchase of most services by most governments agencies. Past experience has shown that a certain government contractor wins, on the average, 3 of every 5 contrasts on which they submit a bid. You want to probability the contractors wins 2 bid submissions out of the 5 the submit.

   f. Computation of the probability that a corporate relations officer receives more than 15 customer complaints during an 8 hours day if it is known that customer complaints arrive randomly and independently at the office at the rate of 10 per 8 hour day.

   g. A particular antibiotic is shipped to drug store in cases, each of which contains 24 bottles. Having doubts about the potency of the drug, the druggist decides to have 5 bottles of the drug tested and reject the case if more than 1 of the bottles is under-strength. Unknown to her, 10 of the 24 bottles are under-strength. What is the probability that she rejects the case.

2. Let X be a Poisson random variable with $\lambda = 2$. Calculate the following probabilities: $P(X = 0)$, $P(X = 1)$, $P(X > 1)$, $P(X \geq 1)$, and $P(1 < X \leq 3)$.

3. It is known that 30% of all calls coming into a telephone exchange are long-distance calls. Suppose 150 calls come into the exchange. Find the probability distribution of the number of long distance call.

4. Nine customers enter a clothing store during a 1 hour period. From past experience, it is known that approximately 30% of the people entering the store make a purchase.

   a. what is the probability that exactly 3 out of 9 customers make a purchase ?

   b. what is the probability that at least one customer make a purchase ?

   c. what is the expectation number of purchase ?

5. A purchaser receives a shipment of 8 computers of which 3 are defective. A random sample of 4 computers is selected and tested. Let X be the number of defective computers selected. Find the probability distribution of X in a table format.

6. During the summer months (June to August inclusive), an average of 5 marriages per month take place in small city. Assuming that these marriages occur randomly and independently of one another, find the probability of the following:

   a. Fewer than 4 marriages will occur in June.

   b. Exactly 10 marriages will occur during the 2 months of July and August.

7. Over the years it has been found that accidents on an assembly line occur randomly and independently at an average rate of 1.5 accidents per week. Find the probability that at least 4 accidents will occur in a 2-week period.

8. A company has five applicants for two positions: two women and three men. Suppose that the five applicants are equally qualified and that no preference is given for choosing either sex. Let X be the number of women chosen to fill the two position.

   a. what is the probability that no women has been selected to fill the two position?

   b. what is the probability that at least one woman has been selected to fill the two positions?

9. A store manager claims that 20% of customers entering the store will make a purchase. Suppose that a random sample of 10 customers is selected.

   a. What is the mean, variance and standard deviation of the number of customers who will make a purchase.

   b. What is the probability that one customer will make a purchase ?

   c. what is the probability that at most two customers will make a purchase ?

10. A small police precinct has 100 residents. The probability that an individual is mugged on a given day is 0.004. If X is a random variable for the number of residents who are mugged on a given day. Find the approximate probability that more than 1 resident is mugged on a given day.

11. The operator on duty at a computer center has observed that print job requests are received according to a Poisson distribution at an average rate of 2 requests every five minutes. The operator take a 15 minute coffee break every morning. What is the probability that the number of print job requests received during his coffee break is less than five ?

12. In a large population of flatworms in a certain pond, 3 in 10 is adult and 7 in 10 is juvenile (Write out the appropriate expressions for the probability).

    a. If 15 flatworms are selected, what is the probability that more than five will be adults ?

    b. Suppose that in another pond, a large population of flatworms exists such that only 1 in 100 is adult. If 400 flatworms are randomly selected from this population, what is the approximate probability that between four and eight, inclusive, are adult? Justify the use of the approximation you use.

13. A batch of 400 resistors is to be shipped if it is found that a random sample of 10 resistors has 2 or fewer defectives. Suppose that there are 40 defectives in the batch of 400 (Write out the appropriate expressions for the probability).

    a. Find the probability that the lot will be shipped.

b. What is the approximate probability that the lot will be shipped ? Justify the use of the approximation you use.

Solutions

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1. (a). Binomial (100, 0.15) (b). Hyper(3, 7, 3) (c). Poisson(4) (d). Hyper(10, 38, 70) (e). Binomial (5, 0.6) (f). Poisson (10) (g). Hyper (5, 10, 14)

2. 0.1353, 0.270, 0.59, 0.864, 0.451

3. $P(x) = \binom{150}{x}(0.3)^x(0.7)^{150-x}$

4. Let X is the number of customers who make a purchase out of 9 $\approx Binomial(9, 0.3)$ (a). $P(3) = 0.2667$ (b). $P(X \geq 1) = 1 - P(X = 0) = 0.9597$ (c). $E(X) = np = 2.7$

5. Hypergeometric random variable with $n = 4, M = 3, N = 8$. $P(X = 0) = \frac{5}{70}$, $P(X = 1) = \frac{30}{70}$, $P(X = 2) = \frac{30}{70}$, $P(X = 3) = \frac{5}{70}$

6. Let X be the number of marriages that occur in June $\approx Poisson(5)$, $P(X < 4) = 0.2650$ (b). Let Y be the number of marriages that occur in July and August $\approx Poisson(10)$, $P(X = 10) = 0.1251$

7. Let X is the number of accidents occur in a 2-week period $\approx Poisson(1.5 \times 2)$. $P(X \geq 4) = 1 - P(X < 4) = 1 - P(X \leq 3) = 0.353$

8. Let X be the number of women chosen to fill the two positions: $\approx Hyper(2, 2, 3)$ (a). $P(X = 0) = \frac{3}{10}$ (b). $P(X \geq 1) = 1 - P(X < 1) = \frac{7}{10}$

9. Let X be the number of customer who make a purchase $\approx Binomial(10, 0.2)$ (a). 2, 1.6, 1.26 (b). $P(X = 1) = 0.2684$(c). $P(X \leq 2) = 0.6778$

10. Let X be the number of residents who are mugged on a given day $\approx Binomial(100, 0.004)$, $np = 4 < 7 \approx Poisson(4)$, $P(X > 1) = 1 - P(X \leq 1) = 0.062$

11. Let X be the number of print job requests received during 15 minute $\approx Poisson(6)$. $P(X < 5) = P(X \leq 4) = 0.285$

12. Let X be the number of adults in 15 samples $\approx Binomial(15, 0.3)$ (a). $P(X > 5) = 1 - P(X \leq 5) = 1 - \sum_{x=0}^{5}\binom{15}{x}(0.3)^x(0.7)^{15-x}$ (b). $n = 400, p = 0.01, np = 4 < 7 \approx Poisson(4)$, $P(4 \leq X \leq 8) = \sum_{x=4}^{8}\frac{4^x e^{-4}}{x!}$

13. Let X be the number of defectives $\approx Hyper(10, 40, 360)$, (a). $P(X \leq 2) = \sum_{x=0}^{2}\frac{\binom{40}{x}\binom{360}{10-x}}{\binom{400}{10}}$ (b). $\frac{n}{N} = 0.025 < 0.05$, $\frac{40}{400} = 0.1 = p$, $Hyper(10, 40, 360) \approx Binomial(10, 0.1)$ $P(X \leq 2) = \sum_{x=0}^{2}\binom{10}{x}(0.1)^x(0.9)^{10-x}$