Exercise 8: Statistics 213 (L05) - Fall 2007

1. If Z is a standard normal random variable, then probability that Z will be greater than -1.26 is equal to

   a. 0.8962 b. 0.3962 c. 0.1038 d. 0.6038 e. none of the above

2. If the random variable Z has a normal distribution with mean 0 and variance 1, then $P(Z < -0.75)$ equals

   a. 0.2266 b. 0.2734 c. 0.7266 d. 0.7734 e. 0.2500

3. The random variable X is normally distributed with mean 1 and variance 4. The probability $P(X = 3.0)$ is equal to

   a. 0 b. 0.1915 c. 0.3413 d. 0.8413 e. 0.6915

4. The random variable X has a normal distribution with standard deviation 8. If $P(X > 25) = 0.9505$, then the mean μ ?

   a. 11.80 b. 29.67 c. 38.20 d. 0 e. not enough information

5. Let the random variable Y be normally distributed with mean 1 and standard deviation 0.5. If $P(Y < C) = 0.03$, the value C equals

   a. 0.0120 b. 0.0300 c. 0.0600 d. 1.8800 e. 1.9400

6. What is the area under the standard normal curve corresponding to $-0.3 < Z < 1.6$?

   a. 0.9542 b. 0.5631 c. 0.3273 d. 0.4713 e. 0.3821

7. What is the 20^{th} percentile of the standard normal distribution ?

   a. 0.84 b. 0.58 c. -0.42 d. -0.84 e. 0.20

8. The 95^{th} percentile of the standard normal probability distribution occurs at the value $z =$

   a. 1.645 b. 1.960 c. 2.054 d. 2.326 e. 2.576

9. If X has the normal distribution with $\mu = 2$ and $\sigma^2 = 40$, then the probability that $X = 3$ is

   a. 0.0000 b. 0.1915 c. 0.0987 d. 0.5987 e. 0.4017

10. The standard normal probability distribution differs from other normal probability distribution in that:

    a. the standard normal probability distribution is not skewed while all others are skewed
    b. the standard normal probability distribution has a standard deviation of zero while all others have standard deviations greater than zero
    c. the standard normal probability distribution is discrete while all others are continous
    d. the standard normal probability distribution can be used to model real-world phenomena while all others are inappropriate since they can take on values from $-\infty$ to ∞.
    e. none of the above is correct

11. The marks on an arithmetic test are normally distributed with a mean of 62 and a variance of 225. If the teacher wishes to assign A's to the top 14% of the students, what minimum mark is required to get an A ?

    a. 78.2 b. 77 c. 67.4 d. 56.6 e. 45.8

12. The weight of food packed in a certain container is normally distributed with a mean of 500 grams and a standard deviation of 5 grams. A random sample of 35 containers is selected and the weight of food in each is measured. The expected number of containers in the sample with less than 498 grams of food is:

    a. 12 b. 5 c. 15 d. 6 e. 8

13. Suppose we sample from a uniform distribution with a =0 and b=10.

    a. What is the probability of obtaining a value between 5 and 7 ?
    b. What is the mean and standard deviation ?

14. A token ring local area network provides an inter-token time having a uniform distribution between 0 and 2 seconds.

    a. Find the probability that the time will be less than 0.6 seconds.
    b. Find the probability that the time will be greater than 1.8 seconds.
    c. Find the probability that the time will between 0.4 and 1.6 seconds.
    d. Find the mean and standard deviation of the inter-token time.

15. A hotel has 160 single rooms. Experience has shown that 10% of the people who make reservation fail to show up. For a particular Friday night, the hotel has made 176 single reservation. Find the probability that a room will be available for all those people with reservations who show up.

16. Mensa is an organization whose members possess IQs in the top 2% of the population. If IQs are normally distributed with a mean of 100 and a variance of 256:

    a. What proportion of the population have IQs between 90 and 120 ?
    b. Find the first and third quartiles, Q_1 and Q_3 for this distribution.
    c. How many standard deviations from the mean are Q_1 and Q_3 ?
    d. What proportion of the population has an IQ less than 80 ?
    e. What is the minimum IQ necessary for admission to Mensa ?
    f. If 5 individuals are chosen at random from the general population, what is the probability that at most 1 of them exceeds the minimum requirement for admission to Mensa ?

17. The lifetime, X, of a particular type of dishwasher is normally distributed with a mean of 10 years and a variance of 4 years.

    a. What is the probability that a dishwasher will last less than 5 years ?
    b. How long a guarantee should the manufacturer offer if he is only willing to replace at most 10% of the dishwashers if they fail during the guarantee period ?

18. Historical data collected at a paper mill reveal that 40% of sheet breaks are due to water drops, which result from the condensation of steam. Suppose that the causes of the next 50 sheet breaks are monitored and that the sheet breaks are independent of one another.

    a. Find the expected value and standard deviation of the number of sheet breaks that will be caused by water drops.

    b. What is the approximate probability that fewer than 25 of the breaks will be due to water drops ?

    c. What is the approximate probability that the number of breaks due to water drops will be between 10 and 25 inclusive ?

Solutions

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1. a 2. a 3. a 4. d 5. c 6. b 7. d 8. a 9. a 10. e 11. a 12. a

13. Let $X \sim Uniform(0, 10)$. a). Want $P(5 < X < 7) = \frac{2}{10} = \frac{1}{5}$, b). $E(X) = \frac{0+10}{2} = 5$, $V(X) = \frac{(10-0)^2}{12} = 8.3333$, $sd(X) = 2.8867$.

14. Let X be the inter-token time $\sim Uniform(0, 2)$. a). Want $P(X < 0.6) = \frac{0.6}{2} = 0.3$ b). Want $P(X > 1.8) = 1 - P(X < 1.8) = 1 - \frac{1.8}{2} = 0.1$ c). Want $P(0.4 < X < 1.6) = P(X < 1.6) - P(X < 0.4) = \frac{1.6-0.4}{2} = \frac{1.2}{2} = 0.6$, d). $E(X) = \frac{0+2}{2} = 1$, $V(X) = \frac{(2-0)^2}{12} = \frac{1}{3} = 0.3333$, and $sd(X) = 0.5773$.

15. Let X be the number who show out of the 176 reservations. $\sim Binomial(176, 0.9)$. Since n: large and np and $n(1-p) > 5$, use Normal with $\mu = 176 \times 0.9$ and $\sigma^2 = 176 \times 0.9 \times 01$. Want $P(X \leq 160) \approx 0.7019$

16. Let X be the IQ score $\sim \mathcal{N}(100, 256)$. a). $P(90 < X < 120) = 0.6301$, b). What Q_1 that satisfies $P(X < Q_1) = 0.25$. $Q_1 = 89.28$. For Q_3, $P(X < Q_3) = 0.75$. $Q_3 = 110.72$, c). 0.67 or $\frac{2}{3}$ since $Q_1 = \mu - 0.67 \times \sigma$ and $Q_3 = \mu + 0.67 \times \sigma$. d). Want $P(X < 80) = 0.1056$, e). Want x_0 such that $P(X > x_0) = 0.02$, $x_0 = 132.88$, f). Let X be the number of people out of sample of 5 whose IQ's exceed the minimum requirement for Mensa $\sim Binomial(5, 0.02)$, Want $P(X \leq 1) = 0.9961$.

17. Let X be the lifetime of a particular type of dishwasher $\sim \mathcal{N}(10, 4)$. a). Want $P(X < 5) = 0.0062$, b). Want x_0 such that $P(X < x_0) = 0.10$, $x_0 = 7.44$. the guarantee should be 7.44 years.

18. Let X be the number of sheet breaks caused by water drops out of the 50 $\sim Binomial(50, 0.4)$. Since $np > 5$ and $n(1-p) > 5$, use Normal with mean np and variance $np(1-p)$. a). $E(X) = np = 20$, b). Want $P(X < 25) = 0.9032$, c). Want $P(10 < X \leq 25) \approx 0.9429$.