

**Exercises : Statistics 213 (L05) - Fall 2007**  
**(Binomial, Sampling Distribution, Inference)**

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1. Your final exam (STAT213 L05) will take place on Wednesday, December 19th, from 12: 00 p.m. to 2:00 p.m., in KN(Kinesiology). You are allowed a non-programable calculator.
  2. There are 27 *multiple choice* and 3 *show your work* questions. You will use 'ScanTron Sheets' for the multiple choice questions and the standard HB pencil is preferred (the scanner will pick up dark blue and black pen as well).
  3. Extended Office Hours: Dec 11(Tuesday), 2:00pm - 4 :00pm, Dec 13 (Thursday), 2:00pm-4:00pm and Dec 18 (Tuesday), 10:00am-11:30am and 2:00pm -4:00pm. Warning: TYPOS !!!
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- topic 1

- observational study v.s experimental study
- population v.s. sample, parameter v.s. statistic
- sampling methods: SRS, systematic sampling, stratified sampling, cluster sampling, multistage sampling
- variables: qualitative (or categorical) variable, quantitative (or numerical) variables (discrete and continuous variables)
- displaying quantitative data: stem-and-leaf plot, histogram (frequency and relative frequency histograms), boxplot
- mean, mode, median, variance, standard deviation, percentile, quartiles, interquartile range
- symmetric, skewed to the right, skewed to the left, outliers (mild and extreme outliers from a boxplot), empirical rule

- topic 2

- scatter plot, correlation coefficient, least squares line, coefficient of determination
- interpretations and predictions

- topic 3 and 4

- sample space, event, probability, additive rule, multiplicative rule, complement rule, permutation, combination, etc
- mutually exclusive, independency , conditional probability, Bayes's rule

- topic 5
  - random variable
  - probability distribution of a random variable
  - joint probability distribution
  - marginal probability distribution
  - conditional probability distribution, independency
  - mean (expected value) and variance of a random variable
  - covariance of random variances, some rules about the mean and variance
- topic 6 and 7
  - discrete random variables: binomial, hypergeometric, and Poisson distributions
  - expectations and variances: expectations, variance and and their linear combinations, approximations
- topic 8 and 7
  - continuous random variables: concept of continuous random variable and its distribution, uniform and Normal distributions, standard Normal distribution (tables), approximation of the binomial by the Normal distribution
- topic 9
  - central limit theorem
  - sampling distributions, distribution of the sample mean from a Normal population, large sample mean distribution, distribution of the sample proportion, introduction to confidence intervals, finding sample size
- topic 10
  - hypothesis testing involving means and proportions with large sample
  - t-distribution (table), confidence interval, sample size, test for the mean with small sample and unknown population variance

**Multiple Choice** questions: Please indicate your selection clearly in the answer sheet provided by shading one of the letters **a**, **b**, **c**, **d**, or **e** corresponding to the correct answer. Each questions is worth 1 mark.

1. A fair die is rolled three times. The probability of at least one 6 is
  - (a)  $\frac{76}{26}$
  - (b)  $\frac{140}{216}$
  - (c)  $\frac{125}{216}$
  - (d)  $\frac{91}{216}$
  - (e) none of the above
  
2. It is known that in a certain large population 20% of the people are afflicted by a particular disease. A research team is to select a random sample of 50 people from the population. If the number of diseased people in the sample is considered as the random variable, the mean and the variance are, respectively.
  - (a) 25 and 8
  - (b) 10 and 2.8
  - (c) 25 and 64
  - (d) 10 and 8
  - (e) none of the above
  
3. The binomial distribution is symmetric when p is equal to
  - (a) 0.25
  - (b) 0.1
  - (c) 0.5
  - (d) not (1-p)
  - (e) none of the above
  
4. If the proportion of left-handed people in a very large population is  $\frac{1}{5}$ , the probability of obtaining exactly 2 left-handed people in a random sample of size 4 is:
  - (a) 0.04
  - (b) 0.0256
  - (c) 0.1531
  - (d) 0.1536
  - (e) 0.3072

5. A professional basketball player sinks 80% of his foul shots, in the long run. If he gets 100 tries during a season, then the probability that he sinks between 75 and 90 shots (inclusive) is approximately equal to
- (a)  $P(-1.25 < Z < 2.5)$
  - (b)  $P(-1.125 < Z < 2.625)$
  - (c)  $P(-1.125 < Z < 2.375)$
  - (d)  $P(-1.375 < Z < 2.625)$
  - (e) none of the above
6. According to genetic theory, the blossom color in the second generation of a certain cross of sweet peas should be red or white in a 3: 1 ratio. That is, each plant has probability  $\frac{3}{4}$  of having red blossoms. The colors of the blossoms on separate plants are determined independently. The probability of getting at least 50 red blossomed plants when 80 plant are grown from seeds is approximately:
- (a) 0.9966
  - (b) 0.9951
  - (c) 0.9929
  - (d) 0.0034
  - (e) 0.4966
7. The distribution of the count,  $X$ , the number of successes in  $n$  trials will be binomial if:
- I. the  $n$  trials are independent
  - II. probability of success on each trial is  $p$
  - III. sampling is done without replacement
- (a) I and II are true
  - (b) I and III are true
  - (c) II and III are true
  - (d) I, II and III are true
  - (e) none of the above
8. According to government data, in a city, 22% of children under the age of 6 live in households with incomes less than the official poverty level. A simple random sample of 300 children is selected. The approximate probability that at least 80 of the children in the sample live in poverty is:
- (a) 0.0256
  - (b) 0.0301

- (c) 0.0423
  - (d) 0.0574
  - (e) 0.0948
9. A normal population has  $\mu = 20$  and standard deviation=5. For which of the following sample sizes will the variance of the sample mean be the largest ?
- (a) 400
  - (b) 5
  - (c) 250
  - (d) 10
  - (e) 1
10. Consider a random sample of size 100 taken from a population with mean=40 and standard deviation =25. The probability that the sample mean is greater than 45 is
- (a) 0.0228
  - (b) 0.0793
  - (c) 0.4207
  - (d) 0.4772
  - (e) 0.2000
11. The central limit theorem tells us that the sampling distribution of  $\bar{X}$  is approximately normal. Which of the following conditions are necessary for the theorem to be valid?
- (a) The sample size has to be large
  - (b) We have to be sampling from a normal population
  - (c) The population has to be symmetric.
  - (d) Population variance has to be small.
  - (e) Both (a) and (c).
12. The central limit theorem is important because, for reasonably large sample sizes,
- (a) it allows us to make inferences concerning the sample mean, whether the population variance is known or not
  - (b) it allows us to make an approximate probability statement concerning the sample mean, without knowledge of the shape of the population distribution
  - (c) it allows us to approximate the distribution of the sample by a t-distribution, in the event that the population variance is unknown
  - (d) it allows us to make probability statements concerning the sample mean, providing the population normal

- (e) it allows us to use the normal distribution to make probability statements concerning the distribution of the sample, providing the population is also normal
13. The precision of an unbiased estimator is measured by its variance. For a random sample of size  $n$  from a normal distribution with mean  $\mu$  and variance  $\sigma^2$ , the sample mean  $\bar{X}$  has a variance equal to:
- (a)  $\frac{\sigma}{\sqrt{n}}$
  - (b)  $n\sigma^2$
  - (c)  $\frac{\sigma^2}{n}$
  - (d)  $\sqrt{n\sigma}$
  - (e)  $\sigma^2$
14. A random sample of size  $n=10$  is drawn from a normal population with mean 75 and variance 90. What is the probability that the sample mean calculated from this sample exceeds 80 ?
- (a) 0.4515
  - (b) 0.2877
  - (c) 0.0485
  - (d) 0.0475
  - (e) none of the above
15. A recycling plant compresses aluminum cans into bales. The weights of the resulting bales are known to follow a distribution with standard deviation =8 pounds. With a simple random sample of 64 bales, what is the probability that the sample mean differs from the population mean by no more than 1 pound ?
- (a) 0.3413
  - (b) 0.4772
  - (c) 0.6826
  - (d) 0.9544
  - (e) 0.1587
16. In a survey of the prevalence of drug use, 400 randomly selected high school students are asked if they have ever used hard drugs. Of these, 50 say that they have. Let  $p$  the probability a high school student has used drug. The standard error of  $\hat{p}$  is:
- (a) 0.00027
  - (b) 0.0165
  - (c) 6.614
  - (d) 0.331

- (e) can't be found from the data given
17. In a study of the effects of acid rain, a random sample of 100 trees from a particular forest are examined. Forty percent of these show some signs of damage. Which of the following statement is correct ?
- (a) The sampling distribution of the proportion of damaged tree is approximately normal.
  - (b) If we took another random sample of trees, we would find that 40% of these would show some signs of damage.
  - (c) If a sample of 1000 trees was examined, the variability of the sample proportion would be larger than for a sample of 100 trees.
  - (d) This is a comparative experiment.
  - (e) none of these
18. A manager who knows no statistics asks you, "What does it mean to say that a production process is in control ? Your reply to the manager's question is:
- (a) The quality of the manufactured products is good.
  - (b) The variable measured has the same probability distribution at all times.
  - (c) The pattern of variation remains stable over time, not that there be no variation in the variable measured.
  - (d) both (a) and (b)
  - (e) both (b) and (c)
19. A manufacture of automobile batteries claims that the distribution of the lengths of life of its best battery has a mean of 54 months and a variance of 36 months. Suppose a consumer group decides to check the claim by purchasing a sample of 50 of these batteries and subjecting them to tests that determine their lives. Assuming the manufacturer's claim is true, what is the probability the consumer group's sample has a mean life of 52 or fewer months ?
- (a) 0.129
  - (b) 0.371
  - (c) 0.009
  - (d) 0.491
  - (e) 0.348
20. The distribution of actual weights of 250 g chocolate bars produced by a certain machine is normal with mean 253 g and standard deviation 3 g. A sample of 5 of these chocolate bars is selected. There is only a 2.5 % chance that the average weight of the sample of 5 of the chocolate bars will be below (                      ) grams.
- (a) 250.4

- (b) 252.6
  - (c) 255.6
  - (d) 258.9
  - (e) not within  $\pm 0.5$  of any of the above
21. A gas utility company wishes to know how much time is needed for the serviceperson to hand a service call. A random sample of 16 gave an average time 62.2 minutes with the standard deviation of 5.0 minutes. The service call time has normally distributed. The 95% confidence interval for the mean service call time is
- (a) (61.534, 62.866)
  - (b) (59.536, 64.846)
  - (c) (61.587, 62.813)
  - (d) (59.550, 64.850)
  - (e) (59.750, 64.650)
22. Suppose we have a random sample of size  $n$  from a normal distribution with unknown mean and unknown variance. Suppose we calculate 92% and 96% confidence intervals for  $\mu$  from this sample. Which of the following is true ?
- (a) 92% confidence interval is shorter than 96% confidence interval
  - (b) 92% confidence interval is wider than 96% confidence interval
  - (c) they are equal because it is the same sample
  - (d) we cannot compare the lengths since the variance is unknown
  - (e) none of the above
23. If (2.5, 4.7) is a 95% confidence interval for  $\mu$ , this means
- (a) the probability that  $\mu$  lies between 2.5 and 4.7 is 0.95
  - (b) the probability that the sample mean lies between 2.5 and 4.7 is 0.95
  - (c) (2.5, 4.7) is in 95 of 100 confidence interval's for  $\mu$  which will include the true value of  $\mu$ .
  - (d) if many random sample (each of the same size) were taken and a 95% confidence interval calculated using each sample, then (in the long run) 95% of these confidence intervals would cover  $\mu$
  - (e) the true value of  $\mu$  lies between 2.5 and 4.7
24. In constructing confidence intervals for the unknown mean of a normal distribution with variance, which of the following statement is false ?
- (a) if the population variance is not known, we use the t-distribution.

- (b) if the population variance is known, as the sample size increase so does the width of the interval.
  - (c) if the population variance is not known, the width of the interval will, on the average, be larger.
  - (d) if the population variance is known, as the confidence coefficient is increased from 95% to 99%, the interval becomes wider for the same sample.
  - (e) if the population variance is known, as the sample size is increased, the interval becomes narrower.
25. Based on random sample of 100 cows of a certain breed, a confidence interval for estimating the true mean yield of milk is given by (41.541, 43.893). If the yields of milk of a cow may be assumed to be normally distributed, with  $\sigma$  equal to 6, what is the level of confidence ?
- (a) 98%
  - (b) 86%
  - (c) 95%
  - (d) 90%
  - (e) 99%
26. You have measured the systolic blood pressure of a random sample of 25 employees of a company. A 95% confidence interval for the mean systolic blood pressure for the employees of this company is (122, 138). Which of the following statements gives a valid interpretation of this interval ?
- (a) 95% of the sample of employees have a systolic blood pressure between 122 and 138
  - (b) 95% of the employees in the company have a systolic blood pressure between 122 and 138
  - (c) If the sampling procedure were repeated many times, then approximately 95% of the resulting confidence intervals would contain the mean systolic blood pressure for employee in the company
  - (d) If the sampling procedure were repeated many times, then approximately 95% of the sample means would be between 122 and 138.
  - (e) The probability that the sample mean falls between 122 and 138 is equal to 0.95.
27. A 95% confidence interval for the mean reading achievement score for a population of third grade student is (74.2, 84.2). The sample mean is
- (a) 74.2
  - (b) 84.2
  - (c) 0.95

- (d) 79.2
- (e) the answer cannot be determined from the information given.
28. A random sample of nine plots had a mean yield of 100 grams and a standard deviation of 15 grams. Find a 98% confidence interval for the unknown population mean yield  $\mu$ . It is known that the yields are normally distributed.
- (a) (88.4, 111.6)
- (b) (90.2, 109.8)
- (c) (85.5, 114.5)
- (d) (88.5, 111.5)
- (e) (91.8, 108.2)
29. A student is interested in estimating the average number of showers per week taken by college students. Based on a preliminary sample he/she believes that  $\sigma^2$  is close to 2.1. How large a sample is needed if his/her estimate is to be within 0.3 with probability of 0.95.
- (a) 183
- (b) 253
- (c) 64
- (d) 359
- (e) 90
30. The blood pressure of adult males is normally distributed with a standard deviation of 5 units. In order to estimate the mean blood pressure to within 2.5 units with a confidence coefficient of 99%, how large a sample is required ?
- (a) 6
- (b) 14
- (c) 16
- (d) 22
- (e) 27
31. It is known from past records that the average weight of hogs 6 weeks after birth is 100 lbs. with standard deviation of 18 lbs. A new diet which is claimed to increase the weight is given to a large group of hogs and a random sample of 36 hogs is selected. The average weight  $\bar{X}$  is found to 107.5. The alternative hypothesis of the test should be:
- (a)  $\mu \neq 100$
- (b)  $\mu < 100$

- (c)  $\mu > 100$   
 (d)  $\bar{X} > 100$   
 (e)  $\bar{X} > 107.5$
32. To determine whether the mean nicotine content of a brand of cigarette is greater than the advertised value of 1.4 mg, a test of
- $$H_0 : \mu = 1.4 \text{ vs } H_a : \mu > 1.4$$
- is conducted with  $\alpha = 0.05$ . The calculated value of the test statistic is  $Z = 2.42$ .
- (a) The result is significant, the p-value is 1.645  
 (b) The result is not significant, the p-value is 1.645  
 (c) The result is significant, the p-value is 0.4922  
 (d) The result is not significant, the p-value is 0.4922  
 (e) The result is significant, the p-value is 0.0078
33. In testing  $H_0 : \mu = 110$  vs  $H_a : \mu \neq 110$  at the 10% level of significant,  $H_0$  is rejected if
- (a) the value of the test statistic lies in the acceptance region  
 (b) the value 110 is contained in the 90% confidence interval  
 (c) the p-value is greater than 0.1.  
 (d) 110 is not contained in the 90% confidence interval  
 (e) 110 is not contained in the 95% confidence interval
34. A significant test gives a  $p$ -value of 0.04. From this we can
- (a) reject  $H_0$  with  $\alpha = 0.01$   
 (b) reject  $H_0$  with  $\alpha = 0.05$   
 (c) say the probability that  $H_0$  is false is 0.04  
 (d) say the probability that  $H_0$  is true is 0.04  
 (e) none of the above
35. Package of frozen peas are supposed to have a mean weight of 10 oz. The manufacturer wishes to detect either too low a mean weight (which is illegal) or too high a weight (which reduces profit). Experience shows that the weights have a normal distribution and that the standard deviation of the population of weights is 0.5 oz even when the mean changes. The approximate null,  $H_0$  and alternative  $H_a$ , hypotheses for the manufacturer's problem would be:
- (a)  $H_0 : \mu = 10\text{oz}$   $H_a : \mu < 10\text{oz}$   
 (b)  $H_0 : \mu \leq 10\text{oz}$   $H_a : \mu > 10\text{oz}$

- (c)  $H_0 : \mu = \bar{x}$   $H_a : \mu \neq \bar{x}$   
(d)  $H_0 : \bar{x} = 10oz$   $H_a : \bar{x} \neq 10oz$   
(e)  $H_0 : \mu = 10oz$   $H_a : \mu \neq 10oz$

Solutions: 1. (d), 2. (d), 3. (c), 4. (d), 5. (d), 6. (a), 7. (a), 8. (b), 9. (e), 10. (a), 11. (a), 12. (b), 13. (c), 14. (d), 15. (c), 16. (b), 17. (a), 18. (e), 19. (c), 20. (a), 21. (b), 22. (a), 23. (d), 24. (b), 25. (c), 26. (c), 27. (d), 28. (c), 29. (e), 30. (e), 31. (c), 32. (e), 33. (d), 34. (b), 35. (e)

**Show your work** questions: Please answer each question in the space provided and it is important to be specific about what you are calculating/drawing, otherwise it is impossible to award partial marks for solutions. If there is not sufficient room, use the back of the page, and state clearly that you are doing so.

1. The amount of time a bank teller spends with each customer has a population mean of 3.10 min and variance 0.16. If a random sample of 64 customers is selected,

- (a) what is the probability that the average time spent per customer will be at least 3 min ?

Solution: Let  $X$ : time spent with a customer,  $\mu = 3.10, \sigma^2 = 0.16, n = 64, (\bar{X}) \sim \mathcal{N}(\mu, \frac{\sigma^2}{n}) = \mathcal{N}(3.10, \frac{0.16}{64})$

$$\text{Want } P(\bar{X} > 3) = P\left(Z > \frac{3-3.10}{\sqrt{\frac{0.16}{64}}}\right) = \dots = 0.9772$$

- (b) There is an 85% chance that the sample mean will be below how many minutes ?

Solution: Want  $x_0$  such that  $P(\bar{X} < x_0) = 0.85$ .

$P(Z < z_0) = 0.85 \implies z_0 = 1.04, \dots, x_0 = 3.152$ . There is an approximate 85% that sample mean is below 3.152.

2. Historical data collected at a paper mill reveal that 40% of sheet breaks are due to water drops, which result from the condensation of steam. Suppose that the causes of the next 50 sheet breaks are monitored and that the sheet breaks are independent of one another.

- (a) Find the expected value and standard deviation of the number of sheet breaks that will be caused by water drops.

Solution:  $X$ : number of sheet breaks caused by water drops out of the 50  $\sim \text{Binomial}(50, 0.4)$

$$E(X) = np = 20, \sigma = \sqrt{np(1-p)} = \sqrt{12} = 3.464$$

- (b) What is the approximate probability that fewer than 25 of the breaks will be due to water drops ?

Solution: Since  $np = 20 > 5, n(1-p) > 5$ , can use Normal with mean 20 variance 3.464

$$\text{Want } P(X < 25) \approx P(X < 24.5) = P\left(\frac{24.5-20}{\sqrt{12}}\right) = \dots = 0.9032$$

- (c) What is the approximate probability that the number of breaks due to water drops will be between 10 and 25 inclusive ?

Solution: Want  $P(10 \leq X \leq 25) \approx P(9.5 \leq X \leq 25.5) = P(-3.03 \leq Z \leq 1.59) = \dots = 0.9429$

3. A case of wine has 12 bottles, 3 of which contain spoiled wine. A sample of 4 bottles is randomly selected from the case.

- (a) What is the probability that the sample contains 2 bottles of spoiled wine ?

Solution: X: number of bottles spoiled in sample of 4 from 12  $\sim Hyper(n = 4, M = 3, N - M = 9)$

$$\text{Want } P(X = 2) = \frac{\binom{3}{2}\binom{9}{2}}{\binom{12}{4}} = \dots = 0.218$$

- (b) What is the probability that all 4 of the sampled bottles are spoiled ?

Solution: Want  $P(X = 4) = 0$  since only 3 spoiled bottles in population

- (c) What are the mean and variance of the number of spoiled bottles in the sample ?

$$\text{Solution: } E(X) = n \left( \frac{M}{N} \right) = 4 \frac{3}{12} = 1, V(X) = n \left( \frac{M}{N} \right) \left( \frac{N-M}{N} \right) \left( \frac{N-n}{N-1} \right) = 4 \frac{3}{12} \frac{9}{12} \frac{8}{11} = 0.5454$$

4. Three data entry specialist enter requisitions into a computer. Specialist A processes 30% of the requisitions, Specialist B processes 45% and Specialist C processes 25%. If 3% of the requisitions entered by A are incorrect, 5% of those entered by B are incorrect, and 2% of those entered by C are incorrect.

- (a) Find the probability that a random requisition was entered incorrectly.

Solution:  $P(A) = 0.3, P(B) = 0.45, P(C) = 0.25$ , Let I be the event a requisition is incorrect,  $P(I|A) = 0.03, P(I|B) = 0.05, P(I|C) = 0.02$

$$P(I) = P(A \cap I) + P(B \cap I) + P(C \cap I) = P(A)P(I|A) + P(B)P(I|B) + P(C)P(I|C) = \dots = 0.0365$$

- (b) If a sample entry is found to be incorrect what is the probability it was entered by Specialist B ?

$$\text{Solution: Want } P(B|I) = \frac{B \cap I}{P(I)} = \dots = 0.616$$

5. Airline passenger arrivals at a passenger screening facility of an airport can be modeled by a Poisson distribution. If passengers arrive at the rate of 10 per minute.

- (a) What is the probability of at least 2 arrivals in a 15-second period ?

Solution: X: number of arrivals in 15-second period  $\sim Poisson(\mu = \frac{10}{60} \times 15 = 2.5)$

$$\text{Want } P(X \geq 2) = 1 - P(X \leq 1) = \dots = 0.594$$

- (b) What is the expected number of arrivals in a 5-second period ?

Solution: X: number of arrivals in 5-second period  $\sim Poisson(\mu = \frac{10}{60} \times 5 = 0.83)$

the expected number is 0.83

6. A significant test was performed to test the null hypothesis  $H_0 : \mu = 2$  vs the alternative hypothesis  $H_a : \mu \neq 2$ . The value of the test statistic is  $Z = 1.40$  based on the standard normal distribution. Find the p-value for this test is approximately.

$$\text{Solution: p-value} = 2P(Z > 1.40) = 2(1 - P(Z < 1.40)) = 2(1 - 0.9192) = 0.1616$$

7. The contents of bottles of beer vary according to a normal distribution with mean 341 ml and standard deviation 3 ml. What is the probability that the mean content of the bottles in a 12 pack of beer is less than 339 ml?

Solution:  $X$ : the content of bottle of beer  $\sim \mathcal{N}(341, 9)$ ,  $n = 12$  (if  $X \sim \mathcal{N}(\mu, \sigma^2)$  with known  $\sigma$ , then  $\bar{X} \sim \mathcal{N}(\mu, \frac{\sigma^2}{n})$  even though the sample size is only 12):  $\bar{X} \sim \mathcal{N}(341, \frac{9}{12})$

Want  $P(\bar{X} < 339)$

$$= P\left(Z < \frac{339 - 341}{\sqrt{\frac{9}{12}}}\right) = P(Z < -2.31) = 0.0104$$

8. A survey conducted to determine, among other things, the cost of vacation taken by single adults in a particular region, 144 individuals were randomly sampled. Each person was asked to assess the total cost of his or her most recent vacation. The average cost was \$2386 and the standard deviation was \$400.

- (a) Estimate with a 99% confidence interval the average cost of a vacation trip for all single adults in the region.

Solution:

the sample size  $n = 144 \geq 30$ , CLT applies.

a 99% confidence interval per the average vacation cost of all single adults in the region is

$$\left(\bar{X} - z_{1-\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}}, \quad \bar{X} + z_{1-\alpha/2} \frac{\hat{\sigma}}{\sqrt{n}}\right) = \left(2386 - 2.575 \frac{400}{\sqrt{144}}, \quad 2386 + 2.575 \frac{400}{\sqrt{144}}\right)$$

$$\vdots$$

$$(2300.17, \quad 2471.83)$$

- (b) How large a sample would have to be taken to estimate the true average cost to within 60 with 99% confidence? Use the sample standard deviation to estimate  $\sigma$ .

Solution:

margin of error  $K = z_{1-\alpha/2} \frac{s}{\sqrt{n}}$

$$60 = 2.575 \times \frac{400}{\sqrt{n}}$$

$\vdots$

$$n = 294.69 \approx 295$$

- (c) If it is known that the cost of vacation has a normal distribution. 25 individuals were sampled and the average cost was \$2386 and the standard deviation was \$400. Estimate with a 99% confidence interval the average cost of a vacation trip for all single adults in the region.

Solution:

the sample size  $n = 25 < 30$ , CLT doesn't apply.

a 99% confidence interval per the average vacation cost of all single adults in the region is

$$\left( \bar{X} - t_{24,0.01/2} \frac{\hat{\sigma}}{\sqrt{n}}, \quad \bar{X} + t_{24,0.01/2} \frac{\hat{\sigma}}{\sqrt{n}} \right) = \left( 2386 - 2.797 \frac{400}{\sqrt{25}}, \quad 2386 + 2.797 \frac{400}{\sqrt{25}} \right)$$

$$\vdots$$

$$(2162.24, \quad 2609.76.)$$

9. The Federal Aviation Administration (FAA) has begun imposing sanctions against airports that fail security tests. At a particular airport, security guards detected only 72 of the 100 mock weapons carried by FAA inspectors. The FAA claims that the detection rate at this airport is well below the national rate of 80%. Test this claim at  $\alpha = 0.05$ .

Solution:

$X$ : number of weapons carried by FAA inspectors.

Hypothesis:  $H_0 : p = 0.8$  vs  $H_a : p < 0.8$

$$\hat{p} = \frac{X}{n} = \frac{72}{100} = 0.72, n\hat{p} > 5, n(1 - \hat{p}) > 5$$

$$\alpha = 0.05$$

Test statistic:

$$Z_{cal} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.72 - 0.8}{\sqrt{\frac{0.8(1-0.8)}{100}}} = \frac{-0.08}{0.04} = -2$$

Rejection region:  $\{Z_{cal} < -z_{\alpha} = -1.645\}$

Reject  $H_0$ , that is, there is sufficient evidence that the detection rate at this airport is well below the national rate of 80%

10. The time to failure (which is normally distributed) is measured for each of 6 items of a new electronic device. The mean and variance of these measurements in hours are found to be 60.4667 and 21.2587 respectively.

- (a) Is there sufficient evidence to indicate that the new device has a mean life longer than 55 hours at the  $\alpha = 0.05$  level ?

Solution:

$X$ : time to failure of an item of the electronics device,  $n = 6, \bar{X} = 60.4667, \hat{\sigma} = 21.2587$

Hypothesis:  $H_0 : \mu = 55$  vs  $H_a : \mu > 55$

$\alpha = 0.05$

Test statistic:

$$t_5 = \frac{\bar{X} - 55}{\frac{\hat{\sigma}}{\sqrt{n}}} = \frac{60.4667 - 55}{\frac{4.6107}{\sqrt{6}}} = 2.904$$

Rejection region:  $\{t_5 > t_{5,0.05} = 2.015\}$

Reject  $H_0$ , that is, the new device has a mean life longer than 55 hours

(b) Find the 95% confidence interval for mean life.

Solution:

the sample size  $n = 6 < 30$ , CLT doesn't apply.

$$\begin{aligned} & \left( \bar{X} - t_{5,0.05/2} \frac{\hat{\sigma}}{\sqrt{n}}, \quad \bar{X} + t_{5,0.05/2} \frac{\hat{\sigma}}{\sqrt{n}} \right) \\ & = \left( 60.4667 - 2.571 \frac{4.6107}{\sqrt{6}}, \quad 60.4667 + 2.571 \frac{4.6107}{\sqrt{6}} \right) \\ & \qquad \qquad \qquad \vdots \\ & \qquad \qquad \qquad (55.63, \quad 65.31) \end{aligned}$$

**End of Examination**

### Formulae

---

$$r = \frac{\hat{\sigma}_{xy}}{\hat{\sigma}_x \hat{\sigma}_y}$$

$$\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = \sum_{i=1}^n x_i y_i - \frac{1}{n} \sum_{i=1}^n x_i \sum_{i=1}^n y_i$$

$$\hat{\beta}_1 = r \times \frac{\hat{\sigma}_y}{\hat{\sigma}_x} = \frac{\sum x_i y_i - \frac{(\sum x_i \sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$


---

$$E(X) = \mu_X = \sum_{all} xP(X = x)$$

$$V(X) = \sum_{all} (x - \mu_X)^2 P(X = x) = \sum_{all} x^2 P(X = x) - \mu_X^2$$

$$P(Y = y) = \binom{n}{x} p^x (1-p)^{n-x}, \quad y = x, 1, 2, \dots, n,$$

$$P(X = x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}, \quad E(X) = n \left( \frac{M}{N} \right), \quad V(X) = n \left( \frac{M}{N} \right) \left( \frac{N-M}{N} \right) \left( \frac{N-n}{N-1} \right)$$

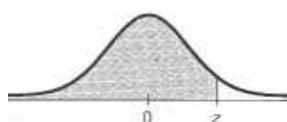
$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, \dots \text{ and } e = 2.7183,$$


---

$$f(x) = P(X = x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b, \quad a \leq b. \\ 0, & \text{otherwise} \end{cases}$$

$$F(x) = P(X \leq x) = \frac{x-a}{b-a}, \quad E(X) = \mu_X = \frac{a+b}{2}, \quad V(X) = \sigma_X^2 = \frac{(b-a)^2}{12}$$

**Table Z (cont.)**  
Areas under the  
standard normal curve

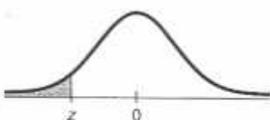


| z   | Second decimal place in z |        |        |        |        |        |        |        |        |        |
|-----|---------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|     | 0.00                      | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
| 0.0 | 0.5000                    | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398                    | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793                    | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179                    | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554                    | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915                    | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257                    | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580                    | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881                    | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159                    | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413                    | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643                    | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849                    | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032                    | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192                    | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332                    | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452                    | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554                    | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641                    | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713                    | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772                    | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821                    | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861                    | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893                    | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918                    | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938                    | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953                    | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965                    | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974                    | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981                    | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987                    | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990                    | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993                    | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995                    | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997                    | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998                    | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998                    | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999                    | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999                    | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |

For  $z \geq 3.90$ , the areas are 1.0000 to four decimal places.

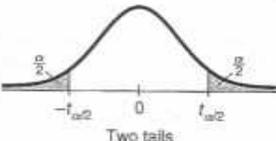
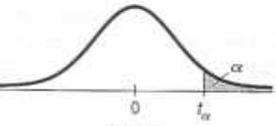
ATH & STATS

**Table Z**  
Areas under the standard normal curve



|        |        | Second decimal place in z |        |        |        |        |        |        |        |        |        |      |
|--------|--------|---------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|------|
|        |        | 0.09                      | 0.08   | 0.07   | 0.06   | 0.05   | 0.04   | 0.03   | 0.02   | 0.01   | 0.00   | z    |
| 0.0001 | 0.0001 | 0.0001                    | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | -3.8 |
| 0.0001 | 0.0001 | 0.0001                    | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | -3.7 |
| 0.0001 | 0.0001 | 0.0001                    | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | -3.6 |
| 0.0002 | 0.0002 | 0.0002                    | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | -3.5 |
| 0.0002 | 0.0003 | 0.0003                    | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | -3.4 |
| 0.0003 | 0.0004 | 0.0004                    | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 | 0.0005 | -3.3 |
| 0.0005 | 0.0005 | 0.0005                    | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0007 | 0.0007 | -3.2 |
| 0.0007 | 0.0007 | 0.0008                    | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 | 0.0009 | 0.0010 | -3.1 |
| 0.0010 | 0.0010 | 0.0011                    | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 | 0.0013 | 0.0013 | -3.0 |
| 0.0014 | 0.0014 | 0.0015                    | 0.0015 | 0.0016 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 | 0.0019 | 0.0019 | -2.9 |
| 0.0019 | 0.0020 | 0.0021                    | 0.0021 | 0.0022 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 | 0.0026 | 0.0026 | -2.8 |
| 0.0026 | 0.0027 | 0.0028                    | 0.0029 | 0.0030 | 0.0031 | 0.0031 | 0.0032 | 0.0033 | 0.0034 | 0.0035 | 0.0035 | -2.7 |
| 0.0036 | 0.0037 | 0.0038                    | 0.0039 | 0.0040 | 0.0041 | 0.0041 | 0.0043 | 0.0044 | 0.0045 | 0.0047 | 0.0047 | -2.6 |
| 0.0048 | 0.0049 | 0.0051                    | 0.0052 | 0.0054 | 0.0055 | 0.0055 | 0.0057 | 0.0059 | 0.0060 | 0.0062 | 0.0062 | -2.5 |
| 0.0064 | 0.0066 | 0.0068                    | 0.0069 | 0.0071 | 0.0073 | 0.0073 | 0.0075 | 0.0078 | 0.0080 | 0.0082 | 0.0082 | -2.4 |
| 0.0084 | 0.0087 | 0.0089                    | 0.0091 | 0.0094 | 0.0096 | 0.0096 | 0.0099 | 0.0102 | 0.0104 | 0.0107 | 0.0107 | -2.3 |
| 0.0110 | 0.0113 | 0.0116                    | 0.0119 | 0.0122 | 0.0125 | 0.0125 | 0.0129 | 0.0132 | 0.0136 | 0.0139 | 0.0139 | -2.2 |
| 0.0143 | 0.0146 | 0.0150                    | 0.0154 | 0.0158 | 0.0162 | 0.0162 | 0.0166 | 0.0170 | 0.0174 | 0.0179 | 0.0179 | -2.1 |
| 0.0183 | 0.0188 | 0.0192                    | 0.0197 | 0.0202 | 0.0207 | 0.0207 | 0.0212 | 0.0217 | 0.0222 | 0.0228 | 0.0228 | -2.0 |
| 0.0233 | 0.0239 | 0.0244                    | 0.0250 | 0.0256 | 0.0262 | 0.0262 | 0.0268 | 0.0274 | 0.0281 | 0.0287 | 0.0287 | -1.9 |
| 0.0294 | 0.0301 | 0.0307                    | 0.0314 | 0.0322 | 0.0329 | 0.0329 | 0.0336 | 0.0344 | 0.0351 | 0.0359 | 0.0359 | -1.8 |
| 0.0367 | 0.0375 | 0.0384                    | 0.0392 | 0.0401 | 0.0409 | 0.0409 | 0.0418 | 0.0427 | 0.0436 | 0.0446 | 0.0446 | -1.7 |
| 0.0455 | 0.0465 | 0.0475                    | 0.0485 | 0.0495 | 0.0505 | 0.0505 | 0.0516 | 0.0526 | 0.0537 | 0.0548 | 0.0548 | -1.6 |
| 0.0559 | 0.0571 | 0.0582                    | 0.0594 | 0.0606 | 0.0618 | 0.0618 | 0.0630 | 0.0643 | 0.0655 | 0.0668 | 0.0668 | -1.5 |
| 0.0681 | 0.0694 | 0.0708                    | 0.0721 | 0.0735 | 0.0749 | 0.0749 | 0.0764 | 0.0778 | 0.0793 | 0.0808 | 0.0808 | -1.4 |
| 0.0823 | 0.0838 | 0.0853                    | 0.0869 | 0.0885 | 0.0901 | 0.0901 | 0.0918 | 0.0934 | 0.0951 | 0.0968 | 0.0968 | -1.3 |
| 0.0985 | 0.1003 | 0.1020                    | 0.1038 | 0.1056 | 0.1075 | 0.1075 | 0.1093 | 0.1112 | 0.1131 | 0.1151 | 0.1151 | -1.2 |
| 0.1170 | 0.1190 | 0.1210                    | 0.1230 | 0.1251 | 0.1271 | 0.1271 | 0.1292 | 0.1314 | 0.1335 | 0.1357 | 0.1357 | -1.1 |
| 0.1379 | 0.1401 | 0.1423                    | 0.1446 | 0.1469 | 0.1492 | 0.1492 | 0.1515 | 0.1539 | 0.1562 | 0.1587 | 0.1587 | -1.0 |
| 0.1611 | 0.1635 | 0.1660                    | 0.1685 | 0.1711 | 0.1736 | 0.1736 | 0.1762 | 0.1788 | 0.1814 | 0.1841 | 0.1841 | -0.9 |
| 0.1867 | 0.1894 | 0.1922                    | 0.1949 | 0.1977 | 0.2005 | 0.2005 | 0.2033 | 0.2061 | 0.2090 | 0.2119 | 0.2119 | -0.8 |
| 0.2148 | 0.2177 | 0.2206                    | 0.2236 | 0.2266 | 0.2296 | 0.2296 | 0.2327 | 0.2358 | 0.2389 | 0.2420 | 0.2420 | -0.7 |
| 0.2451 | 0.2483 | 0.2514                    | 0.2546 | 0.2578 | 0.2611 | 0.2611 | 0.2643 | 0.2676 | 0.2709 | 0.2743 | 0.2743 | -0.6 |
| 0.2776 | 0.2810 | 0.2843                    | 0.2877 | 0.2912 | 0.2946 | 0.2946 | 0.2981 | 0.3015 | 0.3050 | 0.3085 | 0.3085 | -0.5 |
| 0.3121 | 0.3156 | 0.3192                    | 0.3228 | 0.3264 | 0.3300 | 0.3300 | 0.3336 | 0.3372 | 0.3409 | 0.3446 | 0.3446 | -0.4 |
| 0.3483 | 0.3520 | 0.3557                    | 0.3594 | 0.3632 | 0.3669 | 0.3669 | 0.3707 | 0.3745 | 0.3783 | 0.3821 | 0.3821 | -0.3 |
| 0.3859 | 0.3897 | 0.3936                    | 0.3974 | 0.4013 | 0.4052 | 0.4052 | 0.4090 | 0.4129 | 0.4168 | 0.4207 | 0.4207 | -0.2 |
| 0.4247 | 0.4286 | 0.4325                    | 0.4364 | 0.4404 | 0.4443 | 0.4443 | 0.4483 | 0.4522 | 0.4562 | 0.4602 | 0.4602 | -0.1 |
| 0.4641 | 0.4681 | 0.4721                    | 0.4761 | 0.4801 | 0.4840 | 0.4840 | 0.4880 | 0.4920 | 0.4960 | 0.5000 | 0.5000 | -0.0 |

For  $z \leq -3.90$ , the areas are 0.0000 to four decimal places.

|   |          | 0.20  | 0.10  | 0.05   | 0.02   | 0.01   |          |
|---|----------|-------|-------|--------|--------|--------|----------|
| Two tail probability  |          |       |       |        |        |        |          |
| One tail probability  |          | 0.10  | 0.05  | 0.025  | 0.01   | 0.005  |          |
| <b>Table T</b>  |          |       |       |        |        |        |          |
|   | df       |       |       |        |        |        | df       |
| Values of $t_{\alpha}$  |          |       |       |        |        |        |          |
|  |          |       |       |        |        |        |          |
| Two tails   |          |       |       |        |        |        |          |
|  |          |       |       |        |        |        |          |
| One tail  |          |       |       |        |        |        |          |
|   | 1        | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1        |
|   | 2        | 1.886 | 2.920 | 4.303  | 6.965  | 9.925  | 2        |
|   | 3        | 1.638 | 2.353 | 3.182  | 4.541  | 5.841  | 3        |
|   | 4        | 1.533 | 2.132 | 2.776  | 3.747  | 4.604  | 4        |
|   | 5        | 1.476 | 2.015 | 2.571  | 3.365  | 4.032  | 5        |
|   | 6        | 1.440 | 1.943 | 2.447  | 3.143  | 3.707  | 6        |
|   | 7        | 1.415 | 1.895 | 2.365  | 2.998  | 3.499  | 7        |
|   | 8        | 1.397 | 1.860 | 2.306  | 2.896  | 3.355  | 8        |
|   | 9        | 1.383 | 1.833 | 2.262  | 2.821  | 3.250  | 9        |
|   | 10       | 1.372 | 1.812 | 2.228  | 2.764  | 3.169  | 10       |
|   | 11       | 1.363 | 1.796 | 2.201  | 2.718  | 3.106  | 11       |
|   | 12       | 1.356 | 1.782 | 2.179  | 2.681  | 3.055  | 12       |
|   | 13       | 1.350 | 1.771 | 2.160  | 2.650  | 3.012  | 13       |
|   | 14       | 1.345 | 1.761 | 2.145  | 2.624  | 2.977  | 14       |
|   | 15       | 1.341 | 1.753 | 2.131  | 2.602  | 2.947  | 15       |
|   | 16       | 1.337 | 1.746 | 2.120  | 2.583  | 2.921  | 16       |
|   | 17       | 1.333 | 1.740 | 2.110  | 2.567  | 2.898  | 17       |
|   | 18       | 1.330 | 1.734 | 2.101  | 2.552  | 2.878  | 18       |
|   | 19       | 1.328 | 1.729 | 2.093  | 2.539  | 2.861  | 19       |
|   | 20       | 1.325 | 1.725 | 2.086  | 2.528  | 2.845  | 20       |
|   | 21       | 1.323 | 1.721 | 2.080  | 2.518  | 2.831  | 21       |
|   | 22       | 1.321 | 1.717 | 2.074  | 2.508  | 2.819  | 22       |
|   | 23       | 1.319 | 1.714 | 2.069  | 2.500  | 2.807  | 23       |
|   | 24       | 1.318 | 1.711 | 2.064  | 2.492  | 2.797  | 24       |
|   | 25       | 1.316 | 1.708 | 2.060  | 2.485  | 2.787  | 25       |
|   | 26       | 1.315 | 1.706 | 2.056  | 2.479  | 2.779  | 26       |
|   | 27       | 1.314 | 1.703 | 2.052  | 2.473  | 2.771  | 27       |
|   | 28       | 1.313 | 1.701 | 2.048  | 2.467  | 2.763  | 28       |
|   | 29       | 1.311 | 1.699 | 2.045  | 2.462  | 2.756  | 29       |
|   | 30       | 1.310 | 1.697 | 2.042  | 2.457  | 2.750  | 30       |
|   | 32       | 1.309 | 1.694 | 2.037  | 2.449  | 2.738  | 32       |
|   | 35       | 1.306 | 1.690 | 2.030  | 2.438  | 2.725  | 35       |
|   | 40       | 1.303 | 1.684 | 2.021  | 2.423  | 2.704  | 40       |
|   | 45       | 1.301 | 1.679 | 2.014  | 2.412  | 2.690  | 45       |
|   | 50       | 1.299 | 1.676 | 2.009  | 2.403  | 2.678  | 50       |
|   | 60       | 1.296 | 1.671 | 2.000  | 2.390  | 2.660  | 60       |
|   | 75       | 1.293 | 1.665 | 1.992  | 2.377  | 2.643  | 75       |
|   | 100      | 1.290 | 1.660 | 1.984  | 2.364  | 2.626  | 100      |
|   | 120      | 1.289 | 1.658 | 1.980  | 2.358  | 2.617  | 120      |
|   | 140      | 1.288 | 1.656 | 1.977  | 2.353  | 2.611  | 140      |
|   | 180      | 1.286 | 1.653 | 1.973  | 2.347  | 2.603  | 180      |
|   | 250      | 1.285 | 1.651 | 1.969  | 2.341  | 2.596  | 250      |
|   | 400      | 1.284 | 1.649 | 1.966  | 2.336  | 2.588  | 400      |
|   | 1000     | 1.282 | 1.646 | 1.962  | 2.330  | 2.581  | 1000     |
|   | $\infty$ | 1.282 | 1.645 | 1.960  | 2.326  | 2.576  | $\infty$ |
| Confidence levels   |          | 80%   | 90%   | 95%    | 98%    | 99%    |          |

