Stat 213: Intro to Statistics 5

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probability distribution of a random variable X

- a listing, table, or graph that provides the probability $P(X = x)$ associated with each of the value of X

X	x_1	x_2	$\cdots$	x_k	sum
$P(X = x)$	$P(X = x_1)$	$P(X = x_2)$	$\cdots$	$P(X = x_k)$	1

- sum: $\sum_{i=1}^{k} P(X = x_k) = 1$
- $0 \leq P(X = x_i) \leq 1$, for $i = 1, 2, 3, \cdots, k$

example

- X: the sum of upper face number when rolling a pair of dies

X	2	3	4	5	$\cdots$	12	sum
$P(X=x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\cdots$	$\frac{1}{36}$	1

$P(X = 4) = (\qquad\qquad\qquad)$

$P(X \geq 4) = (\qquad\qquad\qquad)$

$P(3 \leq X \leq 5) = (\qquad\qquad\qquad)$

$P(3 < X < 5) = (\qquad\qquad\qquad)$

joint probability distribution

- experiment: a coin is tossed three times
  - $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$
  - X: the number of heads on the first toss
  - Y: the total number of heads
  - the joint probability distribution, $P(X = x, Y = y)$:

	y			
x	0	1	2	3
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

  - $0 \leq P(X = x, Y = y) \leq 1$
  - $\sum_x \sum_y P(X = x, Y = y) = 1$

joint probability distribution

- the joint probability distribution, $P(X = x, Y = y)$:

	y			
x	0	1	2	3
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

$P(X = 0, Y = 1) = \frac{2}{8}$

$P(X = 1, Y = 1) = (\qquad\qquad)$

$P(X = 1, Y = 0) = (\qquad\qquad)$

$P(X = 1, Y = 3) = (\qquad\qquad)$

marginal probability distribution

- the joint probability distribution, $P(X = x, Y = y)$:

	y			
x	0	1	2	3
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

- $P(Y = 0) = P(Y = 0, X = 0) + P(Y = 0, X = 1)$
- $P(Y = 1) = P(Y = 1, X = 0) + P(Y = 1, X = 1)$
- $P(Y = 2) = (\qquad)$
- $P(Y = 3) = (\qquad)$
- $P(X = 0) = (\qquad)$
- $P(X = 1) = (\qquad)$

conditional probability distribution

- the joint probability distribution, $P(X = x, Y = y)$:

	y			
x	0	1	2	3
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

$$P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)}$$

- $P(X = 0|Y = 1) = (\qquad\qquad\qquad\qquad)$
- $P(X = 1|Y = 1) = (\qquad\qquad\qquad\qquad)$

independence

- $P(Y = y|X = x) = P(Y = y)$
- $P(X = x, Y = y) = P(X = x)P(Y = y)$
  - Are $P(X = 1, Y = 1)$ and $P(X = 1)P(Y = 1)$ same ?

mean, variance and standard deviation

- let X be a (discrete) random variable with probability distribution $P(X)$
- the **expected value** of X is given as

$$\mu_X = E(X) = \sum_{\text{all } x} x P(X = x)$$

- the **variance** of X is given as

$$\sigma_X^2 = V(X) = E[(X - \mu_X)^2] = \sum_{\text{all } x} (x - \mu_X)^2 P(X = x)$$

- the **standard deviation** of X is given as

$$\sigma_X = \sqrt{\sigma_X^2}$$

example

X	0	1	2	3	4	5	sum
$P(X=x)$	0.1	0.4	0.2	0.15	0.1	0.05	1

- $\mu_X = E(X) = \sum_{\text{all } x} xP(X=x)$

  $= 0(0.1) + 1(0.4) + 2(0.2) + \cdots + 5(0.05) = 1.9$

- $\sigma_X^2 = V(X) = \sum_{\text{all } x} (x-\mu_X)^2 P(X=x)$

  $= (0-1.9)^2(0.1) + (1-1.9)^2(0.4) + \cdots + (5-1.9)^2(0.05) = 1.79$

- $\sigma_X = \sqrt{\sigma_X^2} = \sqrt{1.79} = 1.34$

some rules

- let X and Y are random variables
  - $E(X + Y + c) = E(X) + E(Y) + c$
  - $E(aX + b) = aE(X) + b$
  - $V(aX + b) = a^2 V(X)$
  - $V(X) = E(X^2) - E(X)^2$

example

- X: the number of heads observed after tossing a coin three times

X	0	1	2	3	sum
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

- $\mu_X = E(X) = (\qquad\qquad\qquad\qquad)$
- $\sigma_X^2 = V(X) = (\qquad\qquad\qquad\qquad)$
- $\sigma_X = (\qquad\qquad\qquad\qquad)$
- $E(2X + 3) = (\qquad\qquad\qquad\qquad)$
- $E(X^2) = (\qquad\qquad\qquad\qquad)$
- $V(2X + 3) = (\qquad\qquad\qquad\qquad)$