Stat 213: Intro to Statistics 6

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example 1

- A box contain 10 black balls and 20 white balls. 5 balls are randomly selected from the box. Let X be the number of black balls in the sample. Suppose that 5 balls are selected one at a time and each time replaced before the next drawn (sampling with `replacement`). What is the probability that 2 black balls are selected in the sample?

  - a Binomial experiment with the the probability of success is $\frac{10}{30}$ where the interest is the number of black balls in the sample: Binomial(n, p) =Binomial $(5, \frac{1}{3})$
  - $P(X = 2) = \binom{5}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^3$
  - $E(X) = np = \frac{5}{3}$
  - $V(X) = np(1 - p) = 5 \times \frac{1}{3} \times \frac{2}{3} = \frac{10}{9}$

example 2

- A box contain 10 black balls and 20 white balls. 5 balls are randomly selected from the box `without replacement`. Let X be the number of black balls in the sample.
  - `NOT` a Binomial experiment

$P(\text{success on the 1st trial}) = \frac{10}{30}$

$P(\text{success on the 2nd trial} \mid \text{success on the 1st trial}) = \frac{9}{29}$

$P(\text{success on the 2nd trial} \mid \text{failure on the 1st trial}) = \frac{10}{29}$

the Hypergeometric random variable

- Consider an experiment of randomly drawing n elements `without replacement` from a set of N elements, M of which are `successes` and $N - M$ of which are `failures`. A random variable, X is the `number of successes` in the draw of n elements.

example 2

- A box contain 10 black balls and 20 white balls. 5 balls are randomly selected from the box `without` replacement. What is the probability that 2 black balls are selected in the sample?

the Hypergeometric distribution: Hyper(n, M, N-M)

- Corresponds to Binomial sampling without replacement
- A population contains M success and $N-M$ failures. The probability of exactly k successes in a random sample of size n is

$$P(X=x)=\frac{\begin{pmatrix} M \\ x \end{pmatrix}\begin{pmatrix} N-M \\ n-x \end{pmatrix}}{\begin{pmatrix} N \\ n \end{pmatrix}}$$

- $E(X)=n\left(\frac{M}{N}\right)$
- $V(X)=n\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-n}{N-1}\right)$
- $\frac{n}{N}\geq 0.05$: the number in population is small in relation to the sample size, the probability of success for a given trial is dependent on the outcomes of preceding trials

example 2

- Let X be the number of black balls in the sample.
- $N = 30,\ M = 10,\ N - M = 20,\ n = 5$
  - $P(X = 3) =$
- $E(X) = n\left(\frac{M}{N}\right) =$
- $V(X) = n\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-n}{N-1}\right) =$

example 3

- A purchaser receives a shipment of $100,000$ computer chips of which $5\%(5000)$ are known to be defective. A random sample of 8 chips is selected and tested. Let X be the number of defective chips in the sample of 8.
  - $N = 100,000,\ M = 5000,\ n = 8,\ \frac{n}{N} = \frac{8}{100{,}000} < 0.05$

- the Binomial distribution can be used to approximate the hypergeometric probabilities in the situation of sampling without replacement from a finite population.
  - $n = 8, \ \ p = \frac{M}{N} = \frac{5000}{100{,}000} = 0.005$

example 4

- A purchase receives a shipment of 8 computers of which 3 are defective. A random sample of 4 computers is selected and tested. Let X be the number of defective computers selected.
  - Find the probability distribution for the number of defective computers.
  - Find mean and standard deviation for the number of defective computers.