Stat 213: Intro to Statistics 6

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Fall 2007

examples

- the number of machine breakdowns during a day
- the number of traffic accidents on a certain section of highway during a one-week period
- the number of calls received by a switchboard during a given period of time
- the number of customer arrives at a checklist counter during given minute

Poisson random variable

- We are counting the number of occurrences of rare events: that is, events whose probability of occurrence is very small.
- The Poisson distribution can be used to model the number of occurrences of a rare event over time, area, etc.

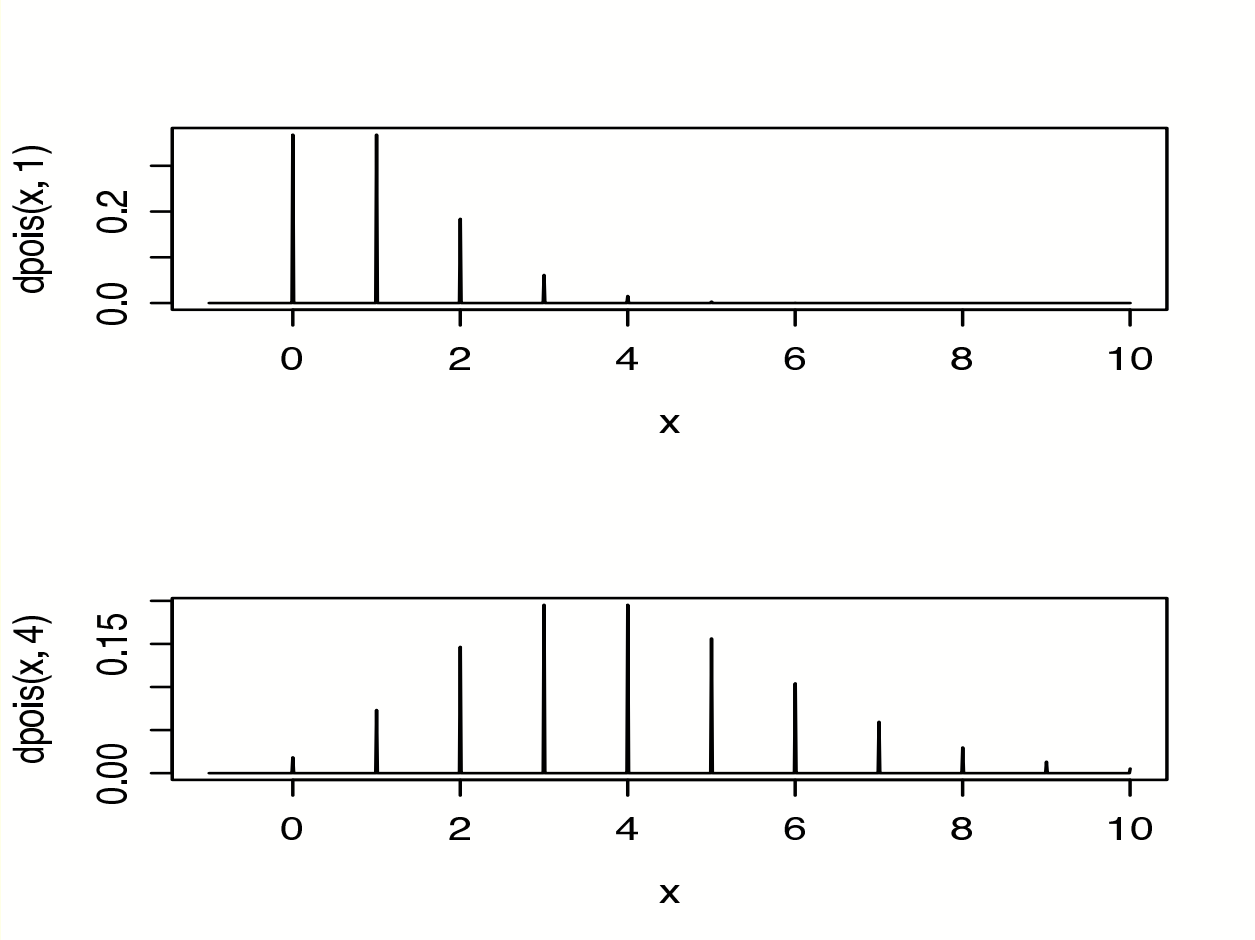
the Possion distribution: Poisson(λ)

- Provides a good model for the data the present the number of events occurring in a period of time or a region of space
- The probability of exactly x occurrences of the event is

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \ x = 0, 1, 2, 3, \cdots \text{ and } e = 2.7183$$

- $E(X) = \lambda$: the average number of occurrences in a certain period of time, space, etc
- $V(X) = \lambda$
- standard deviation $= \sqrt{\lambda}$

shape of the Poisson distribution



example 1

- A typist average 2.5 errors per page.
  a. What is the probability that on a particular page the typist makes four errors?

  b. What is the expected number of errors the typist would make on a 2-page?

  c. What is the probability that the typist makes fewer than 4 errors in 2 pages?

example 2

- Official records in a particular city show that the average number of school closing in a school year due to snowstorms is 4

  a. what is the probability that there will be 6 school closing this year due to snowstorms ?

  b. what is the probability that there will be 16 closing during the next 3 years due to snowstorms ?

example 3

- Airline passengers arrive randomly and independently at the passenger-sceening facility of a major international airport at an average rate of 10 passengers per minute.
  a. what is the probability of at least 2 arrivals in a 15 second period?

  b. what is the expected number of arrivals in a 5-min period?

Poisson approximation to Binomial distribution

- Suppose that
  - an experiment consists of a very large number of independent trials, and
  - at each trial, the rare event has a small probability p of occurring, and a large probability of NOT occurring.
- When n is large and $np < 7$, we may use a Poisson distribution with mean np to approximate the Binomial distribution

example 4

- Suppose that 1% of all transistors produced by a certain company is defective. If 500 transistors are selected at random from the company's assembly line.

  a. Find the exact probability that the sample will contain 3 defectives.

  b. Use Poisson approximation to get answer of (a).