Stat 213: Intro to Statistics 6

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example

- tossing a coin
- X: random variable -number of success

$$X = \begin{cases} 1 & \text{if success with probability, } \frac{1}{2} \\ 0 & \text{if failure with probability, } 1 - \frac{1}{2} = \frac{1}{2} \end{cases}$$

- probability distribution

X	0	1	total
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{2}$	1

- $E(X) = 0 \times (\frac{1}{2}) + 1 \times \frac{1}{2} = \frac{1}{2}$
- $V(X) = E(X^2) - E(X)^2 = [0^2 \times \frac{1}{2} + 1^2 \times \frac{1}{2}] - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Bernoulli trial

- an experiment that are only two events of interest (success and failure) on a single trial
- X: random variable -number of successes

$$X = \begin{cases} 1 & \text{if success with probability, } p \\ 0 & \text{if failure with probability, } 1-p \end{cases}$$

- probability distribution

X	0	1	total
$P(X=x)$	$1-p$	p	1
	$=p^0(1-p)^1$	$=p^1(1-p)^0$	1

Bernoulli trial

X	0	1	total
$P(X=x)$	$p^0(1-p)^1$	$p^1(1-p)^0$	1

$$P(X=x)=p^x(1-p)^{1-x}, \quad x=0,1$$

- $E(X)=(\qquad\qquad\qquad\qquad\qquad\qquad)$
- $V(X)=(\qquad\qquad\qquad\qquad\qquad\qquad)$

example

- Consider the spin of a roulette wheel. A gambler places a bet that the little white ball will land on a 'black' pocket. Defining a random variable X as the outcome from this wager, one can consider this as a Bernoulli trial: a success will occur if the gambler wins the bet- the white ball lands in the 'black' pocket, a failure as the white ball dose not land in a 'black' pocket. In this instance, we can give the probability distribution of the random variable X, where $p = P(X = 1) = \frac{18}{38}$:

X	0	1	total
$P(X = x)$	0.5263	0.4737	1

We can see that the expected value of X is 0.4737 and the standard deviation of X is 0.4993.

the Binomial probability distribution

- Binomial distribution is based on the idea of Bernoulli trial
  - 100 times tossing a coin (head and tail)
- interest in the number of successes after n trials
  - want to know the probability of getting a certain number of successes in a given number of trials

properties

- there are identical trials
- the trials are independent
- each trial results in one of two outcome: success or failure
- the probability of success is the same on each trial, call it p

example

- the probability distribution of obtaining the face five on three rolls of a die
  - X: number of the face five

  - $P(X=0) = \left(\frac{1}{6}\right)^0 \left(\frac{1}{6}\right)^3 = \frac{125}{216}$
  - $P(X=1) = \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^1 \left(\frac{1}{6}\right)^2 = \frac{75}{216}$
  - $P(X=2) = \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^1 + \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^1 + \left(\frac{1}{6}\right)^2 \left(\frac{1}{6}\right)^1 = \frac{15}{216}$
  - $P(X=3) = \left(\frac{1}{6}\right)^3 \left(\frac{1}{6}\right)^0 = \frac{1}{216}$

  - $E(X) = 0 \times \frac{125}{216} + 1 \times \frac{75}{216} + 2 \times \frac{15}{216} + 3 \times \frac{1}{216} = \frac{108}{216} = \frac{1}{2}$
  - $V(X) = (\qquad\qquad\qquad\qquad\qquad\qquad)$

recall: Bernoulli trial

- an experiment that are only two events of interest (success and failure) on a single trial
- X: random variable -number of successes

$$X = \begin{cases} 1 & \text{if success with probability, } p \\ 0 & \text{if failure with probability, } 1-p \end{cases}$$

- probability distribution

X	0	1	total
$P(X=x)$	$p^0(1-p)^1$	$p^1(1-p)^0$	1

two independent Bernoulli trials

- $Y = X_1 + X_2$: random variable -number of successes

$$X_i = \begin{cases} 1 & \text{if success with probability, } p \\ 0 & \text{if failure with probability, } 1-p \end{cases} \qquad i = 1, 2$$

- $Y = 0 \Longrightarrow (X_1, X_2) = (0, 0)$
  - $P(Y = 0) = p^0(1-p)^2 = (1-p)^2$
- $Y = 1 \Longrightarrow (X_1, X_2) = (1, 0)$ or $(0, 1)$
  - $P(Y = 1) = p^1(1-p)^1 + p^1(1-p)^1 = 2p(1-p) = \binom{2}{1}p(1-p)$
- $Y = 2 \Longrightarrow (X_1, X_2) = (1, 1)$
  - $P(Y = 2) = p^2(1-p)^0 = p^2$

two independent Bernoulli trials

- $Y = X_1 + X_2$: number of successes
- probability distribution:

$$P(Y = y) = \binom{2}{y} p^y (1-p)^{2-y}, \; y = 0, 1, 2$$

Y	0	1	2	total
$P(Y = y)$	$(1-p)^2$	$2p(1-p)$	p^2	1

- $E(Y) = (\qquad)$
- $V(Y) = (\qquad)$

n `Bernoulli trials:` $Y \sim$ `Binomial (n, p)`

- $Y = X_1 + X_2 + \cdots + X_n$: random variable -number of successes
- probability distribution:

$$P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}, \ y = 0, 1, 2, \cdots, n$$

- $E(Y) = \mu_Y = \sum_{\text{all } y} yP(Y = y) = (\qquad\qquad\qquad\qquad)$
- $V(Y) = \sigma_Y^2 = \sum_{\text{all } y} (y - \mu_Y)^2 P(Y = y) = (\qquad\qquad\qquad\qquad)$

example

- in the previous example
  - X: number of the face five
  - X: Binomial $(3, \frac{1}{6})$

$$P(X = x) = \binom{3}{x} \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{3-x}, \; y = 0, 1, 2, 3$$

  - $P(X = 0) = (\qquad\qquad\qquad\qquad)$
  - $P(X = 1) = (\qquad\qquad\qquad\qquad)$
  - $P(X = 2) = (\qquad\qquad\qquad\qquad)$
  - $P(X = 3) = (\qquad\qquad\qquad\qquad)$

  - $E(X) = np = \frac{3}{6} = \frac{1}{2}$
  - $V(X) = np(1 - p) = 3 \times \frac{1}{6} \times \frac{5}{6} = \frac{15}{36}$

example

- A student is taking a multiple choice exam that consists of 5 questions. Each question has 4 possible answers. He/She answers every questions. What is the probability that he/she passes the exam if he/she needs at least 4 correct answers to pass ?

shape of Binomial distributions

- the probability distribution of obtaining the face five on three rolls of a die
  - X: number of the face five $= 0, 1, 2, 3$: Binomial $(3, \frac{1}{6})$

shape of Binomial distributions

- the probability distribution of obtaining heads on three tosses of a fair coin
  - X: number of heads $= 0, 1, 2, 3$: Binomial $(3, \frac{1}{2})$

shape of Binomial distributions

- the probability distribution of obtaining heads on four tosses of a fair coin
  - X: number of the face five $= 0, 1, 2, 3, 4$: Binomial $(4, \frac{1}{2})$