Stat 213: Intro to Statistics 8

H. Kim

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continuous random variable

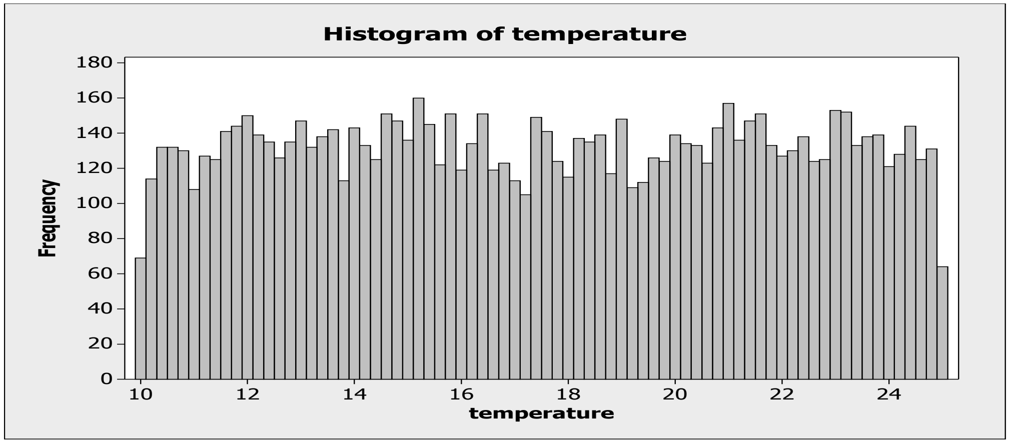
- Continuous random variables can assume the infinitely many values corresponding to points on a line interval
- If you try to assign a probability, to each of these uncountable values, the probability will no longer sum to 1, as with discrete random variables

probability density function

- We have the depth or density of the probability, which varies with x, may be described by a mathematical formula, $f(x)$, called the probability density function for the random variable X.
- The density function $f(x)$ represents the population relative frequency histogram of the continuous random variable.

example

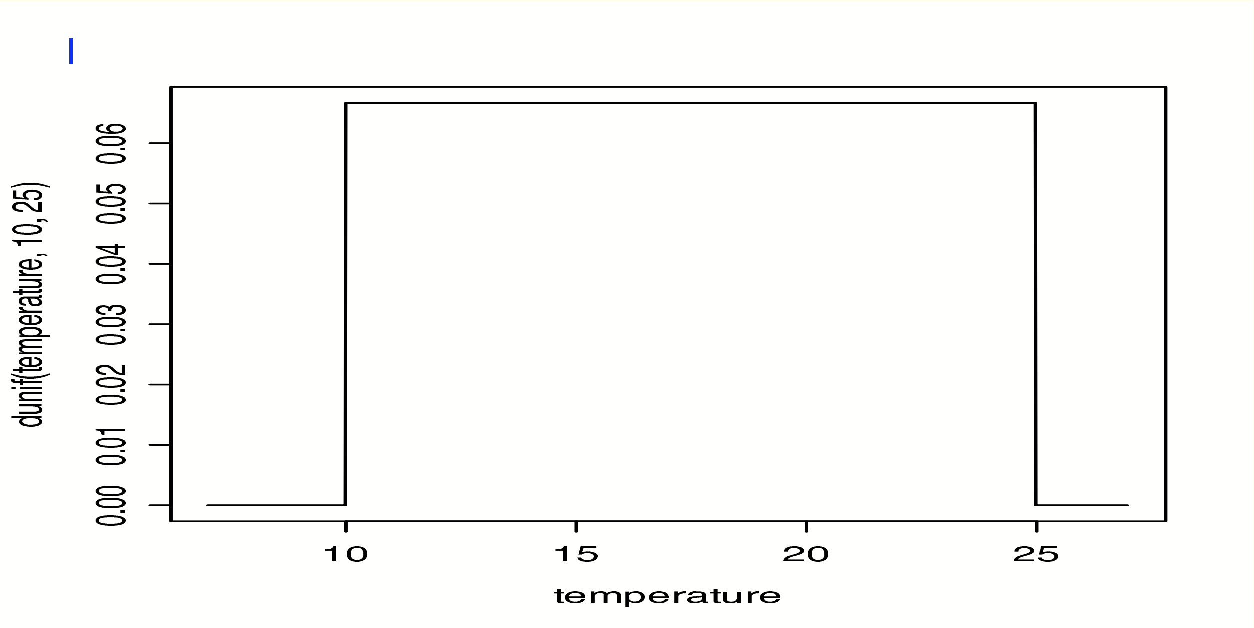
- At an ocean-side nuclear power plant, see water is used as part of the cooling system. This use raises the temperature of the water that is discharged back into the ocean. The amount of the water temperature is raised between 10^oC and 25^oC.



  - What is the probability that the temperature increase will be less than 20 ?

example

- At an ocean-side nuclear power plant, see water is used as part of the cooling system. This use raises the temperature of the water that is discharged back into the ocean. The amount of the water temperature is raised between $10^o C$ and $25^o C$.



the Uniform probability distribution: Uniform(a, b)

- probability density function:

  $$f(x) = P(X = x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b,\ a \leq b. \\ 0, & \text{otherwise} \end{cases}$$

  - the **area** under a continuous probability distribution = 1
  - the probability that X will fall into a particular interval is equal to the **area** under the curve on the interval

  - $P(X = a) = 0$ for continuous random variables
  - $P(X \geq a) = P(X > a)$ or $P(X \leq a) = P(X < a)$

the Uniform probability distribution: Uniform(a, b)

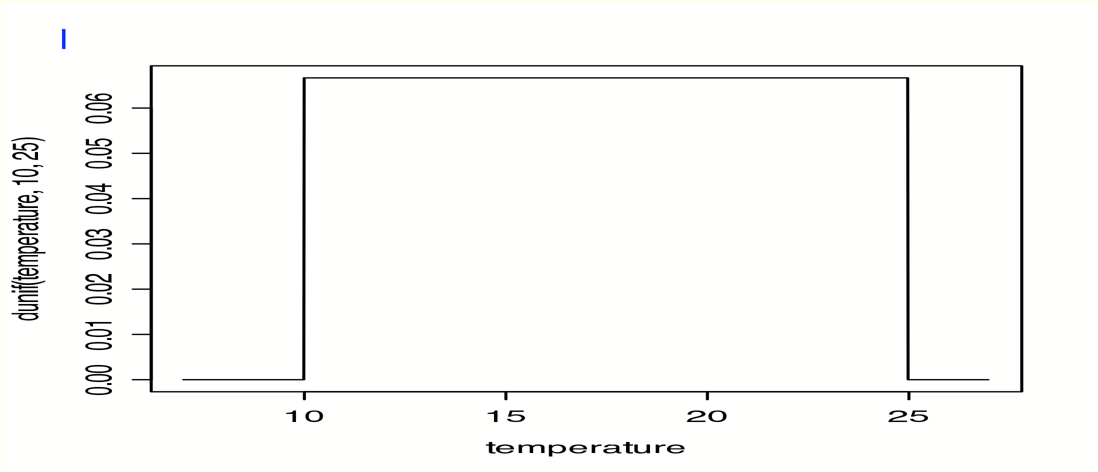
- probability density function:

$$f(x) = P(X = x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b,\ a \leq b. \\ 0, & \text{otherwise} \end{cases}$$

  - cumulative probability density function:
    $F(x) = P(X \leq x) = \frac{x-a}{b-a}$
  - $E(X) = \mu_X = \frac{a+b}{2}$
  - $V(X) = \sigma_X^2 = \frac{(b-a)^2}{12}$

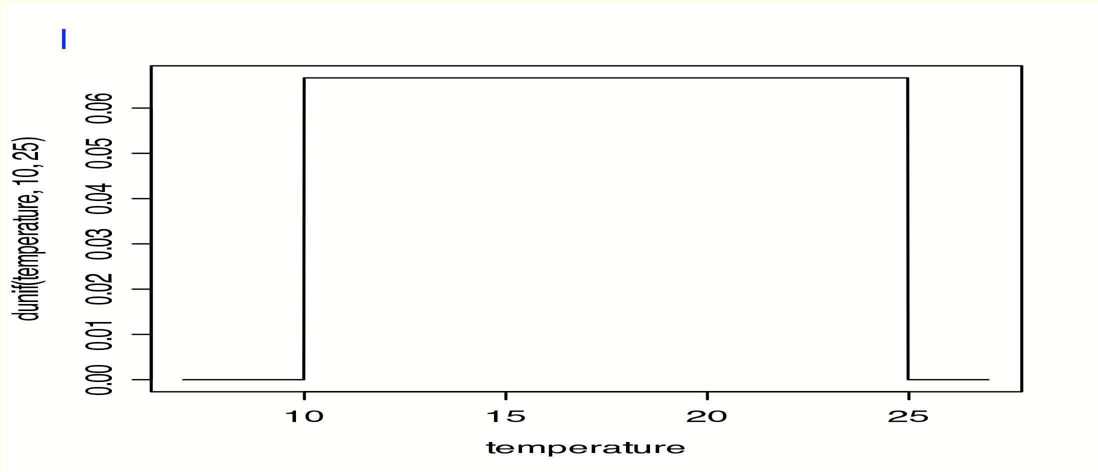
example

- The amount that the water temperature is raised has a uniform distribution over the interval from $10^o C$ to $25^o C$: $\texttt{Uniform}(10, 25)$
  - What is the probability that the temperature increase will be less than $20^o C$?



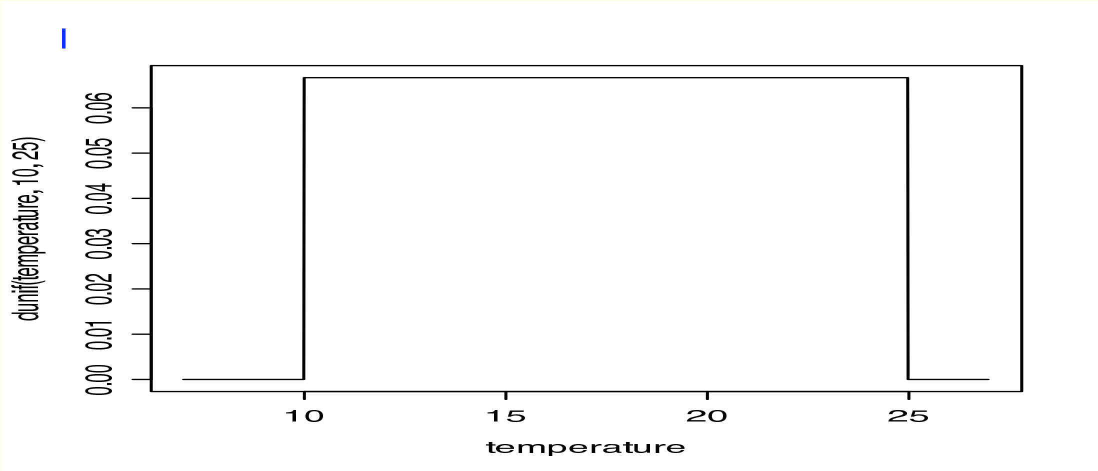
example

- The amount that the water temperature is raised has a uniform distribution over the interval from 10^oC to 25^oC: Uniform$(10, 25)$
  - What is the probability that the temperature increase will be between 15^oC and 18^oC ?



`example: Uniform`$(10, 25)$

– The probability that the water temperature increase is less than this amount is 0.5. What is the amount ? i.e find the 50^{th} percentile of the distribution, or find x_0 such that $P(X < x_0) = 0.5$.



`example:` `Uniform`$(10, 25)$

– What is the mean and the standard deviation of the temperature increases ?