STAT 213, Section 05

Solutions to Assignment #2

11.14 a.

x_i	y_i	x_i^2	x_iy_i
7	2	$7^2 = 49$	$7(2) = 14$
4	4	$4^2 = 16$	$4(4) = 16$
6	2	$6^2 = 36$	$6(2) = 12$
2	5	$2^2 = 4$	$2(5) = 10$
1	7	$1^2 = 1$	$1(7) = 7$
1	6	$1^2 = 1$	$1(6) = 6$
3	5	$3^2 = 9$	$3(5) = 15$

Totals:
$$\sum x_i = 7+4+6+2+1+1+3 = 24$$
$$\sum y_i = 2+4+2+5+7+6+5 = 31$$
$$\sum x_i^2 = 49+16+36+4+1+1+9 = 116$$
$$\sum x_iy_i = 14+16+12+10+7+6+15 = 80$$

b. $$SS_{xy} = \sum x_iy_i = \frac{\left(\sum x_i\right)\left(\sum y_i\right)}{n} = 80 - \frac{(24)(31)}{7} = 80 - 106.2857143 = -26.2857143$$

c. $$SS_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{7} = 116 - \frac{(24)^2}{7} = 116 - 82.28571429 = 33.71428571$$

d. $$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{-26.2857143}{33.71428571} = -.779661017 \approx -.7797$$

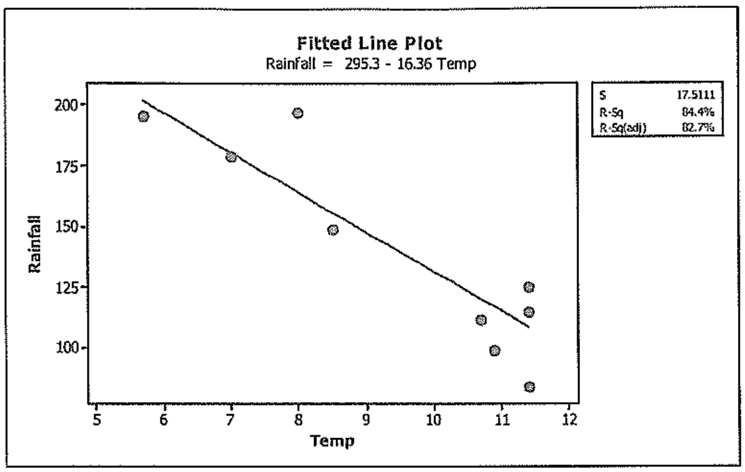
e. $$\bar{x} = \frac{\sum x_i}{n} = \frac{24}{7} = 3.428571429 \qquad \bar{y} = \frac{\sum y_i}{n} = \frac{31}{7} = 4.428571429$$

f. $$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1\bar{x} = 4.428571429 - (-.779661017)(3.428571429)$$
$$= 4.428571429 - (-2.673123487) = 7.101694916 \approx 7.102$$

g. The least squares line is $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 7.102 - .7797x$.

11.22 a. From the printout, the least squares prediction equation is $\hat{y} = 295.25 - 16.364x$.

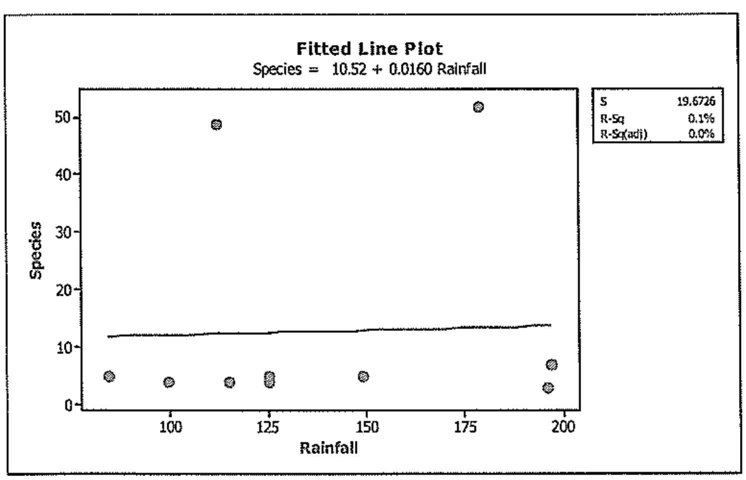
b. Using MINITAB, the fitted regression plot and scatterplot are:



Since the data are fairly close the least squares prediction line, the line is a good predictor of annual rainfall.

c. From the printout, the least squares prediction equation is $\hat{y} = 10.52 + .016x$

Using MINITAB, the fitted regression plot and scatterplot are:



Since the data are not close to the least squares prediction line, the line is not a good predictor of ant species.

11.24 a. Some preliminary calculations are:

$\sum x_i = 62$ $\sum y_i = 97.8$ $\sum x_i y_i = 1{,}087.78$

$\sum x_i^2 = 720.52$ $\sum y_i^2 = 1{,}710.2$

$$SS_{xy} = \sum x_i y_i - \frac{\sum x_i \sum y_i}{n} = 1{,}087.78 - \frac{62(97.8)}{6} = 77.18$$

$$SS_{xx} = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = 720.52 - \frac{62^2}{6} = 79.8533333$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{77.18}{79.8533333} = .966521957$$

$$\hat{\beta}_o = \bar{y} - \hat{\beta}_1 \bar{x} = \frac{97.8}{6} - .966521957\left(\frac{62}{6}\right) = 6.312606442$$

The least squares prediction equation is $\hat{y} = 6.313 + .9665x$

a. The y-intercept is 6.313. This value has no meaning because 0 is not in the observed range of the independent variable mean pore diameter.

b. The slope of the line is .9665. For each unit increase in mean pore diameter, the mean porosity is estimated to increase by .9665.

c. For $x = 10$, $\hat{y} = 6.313 + .9665(10) = 15.978$.

11.70 From Exercises 11.14 and 11.37,

$$r^2 = 1 - \frac{\text{SSE}}{\text{SS}_{yy}} = 1 - \frac{1.22033896}{21.7142857} = 1 - .0562 = .9438$$

94.38% of the total sample variability around $\bar{y}$ is explained by the linear relationship between y and x.

(11.72) a. Some preliminary calculations are:

$$\sum x = 0 \qquad \sum x^2 = 10 \qquad \sum xy = 20$$
$$\sum y = 12 \qquad \sum y^2 = 70$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 20 - \frac{0(12)}{5} = 20$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 10 - \frac{0^2}{5} = 10$$

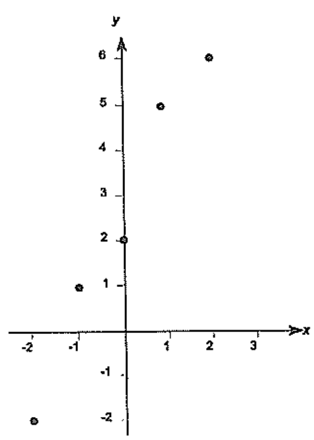
$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 70 - \frac{12^2}{5} = 41.2$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx} SS_{yy}}} = \frac{20}{\sqrt{10(41.2)}} = .9853$$

$$r^2 = .9853^2 = .9709$$

Since $r = .9853$, there is a very strong positive linear relationship between x and y.

Since $r^2 = .9709$, 97.09% of the total sample variability around $\bar{y}$ is explained by the linear relationship between x and y.



b. Some preliminary calculations are:

$$\sum x = 0 \qquad \sum x^2 = 10 \qquad \sum xy = -15$$
$$\sum y = 16 \qquad \sum y^2 = 74$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = -15 - \frac{0(16)}{5} = -15$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 10 - \frac{0^2}{5} = 10$$

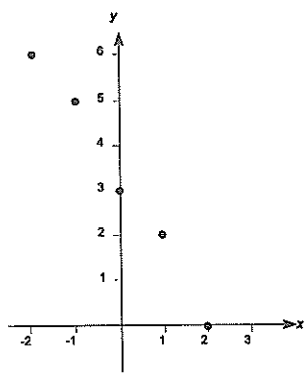
(continued)

11.72 (continued)

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 74 - \frac{16^2}{5} = 22.8$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{-15}{\sqrt{10(22.8)}} = -.9934$$

$$r^2 = (-.9934)^2 = .9868$$



Since $r = -.9934$, there is a very strong negative linear relationship between x and y.

Since $r^2 = .9868$, 98.68% of the total sample variability around $\bar{y}$ is explained by the linear relationship between x and y.

c. Some preliminary calculations are:

$$\sum x = 18 \qquad \sum x^2 = 52 \qquad \sum xy = 36$$

$$\sum y = 14 \qquad \sum y^2 = 32$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 36 - \frac{18(14)}{7} = 0$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 52 - \frac{18^2}{7} = 5.71428571$$

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 32 - \frac{14^2}{7} = 4$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{0}{\sqrt{5.71428571(4)}} = 0$$

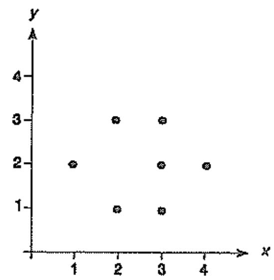
$$r^2 = 0^2 = 0$$

Since $r = 0$, this implies that x and y are not related.

(continued)

11.72 (continued)

Since $r^2 = 0$, 0% of the total sample variability around $\bar{y}$ is explained by the linear relationship between x and y.



d. Some preliminary calculations are:

$$\sum x = 15 \qquad \sum x^2 = 71 \qquad \sum xy = 12$$
$$\sum y = 4 \qquad \sum y^2 = 6$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 12 - \frac{15(4)}{5} = 0$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 71 - \frac{15^2}{5} = 26$$

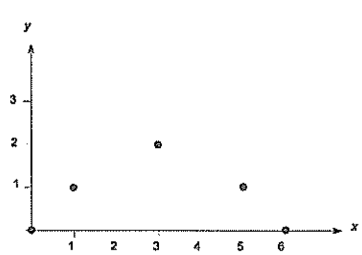
$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 6 - \frac{4^2}{5} = 2.8$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{0}{\sqrt{26(2.8)}} = 0$$

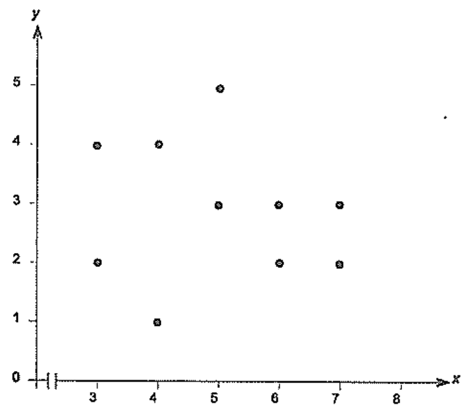
$$r^2 = 0^2 = 0$$

Since $r = 0$, this implies that x and y are not related.

Since $r^2 = 0$, 0% of the total sample variability around $\bar{y}$ is explained by the linear relationship between x and y.



11.110 a.



b. Some preliminary calculations are:

$$\sum x = 50 \qquad \sum x^2 = 270 \qquad \sum xy = 143$$
$$\sum y = 29 \qquad \sum y^2 = 97$$

$$SS_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 143 - \frac{50(29)}{10} = -2$$

$$SS_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 270 - \frac{50^2}{10} = 20$$

$$SS_{yy} = \sum y^2 - \frac{\left(\sum y\right)^2}{n} = 97 - \frac{29^2}{10} = 12.9$$

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx}SS_{yy}}} = \frac{-2}{\sqrt{20(12.9)}} = -.1245$$

$$r^2 = (-.1245)^2 = .0155$$

3.10 a. If the simple events are equally likely, then

$$P(1) = P(2) = P(3) = \cdots = P(10) = \frac{1}{10}$$

Therefore,

$$P(A) = P(4) + P(5) + P(6) = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{3}{10} = .3$$

$$P(B) = P(6) + P(7) = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = .2$$

b. $$P(A) = P(4) + P(5) + P(6) = \frac{1}{20} + \frac{1}{20} + \frac{3}{20} = \frac{5}{20} = .25$$

$$P(B) = P(6) + P(7) = \frac{3}{10} + \frac{3}{10} = \frac{6}{10} = .3$$

3.14 a. The sample space for this experiment would consist of pairs of digits, indicating the result on each of the two dice.

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

b. Each of the above sample points are equally likely, and each therefore has a probability of 1/36.

c. The probability of each event below can be obtained by counting the number of sample points which belong to the event and multiplying this amount by 1/36. This results in the following:

$$P(A) = \frac{1}{36} \qquad P(B) = \frac{18}{36} \qquad P(C) = \frac{6}{36}$$

$$P(D) = \frac{11}{36} \qquad P(E) = \frac{6}{36}$$

3.15 (a) Label the 5 marbles as B_1, B_2, R_1, R_2, R_3.

The sample space consists of $\binom{5}{2} = 10$ unordered pairs.

$$S = \{B_1B_2, B_1R_1, B_1R_2, B_1R_3, B_2R_1, B_2R_2, B_2R_3, R_1R_2, R_1R_3, R_2R_3\}$$

(b) The prob of each sample point is 1/10.

(c) $P[2 \text{ blue marbles}] = P\{B_1B_2\} = 1/10$

$$P\{1 \text{ red and } 1 \text{ blue}\} = P\{B_1R_1 \cup B_1R_2 \cup B_1R_3 \cup B_2R_1 \cup B_2R_2 \cup B_2R_3\} = 6/10$$

$$P[2 \text{ red marbles}] = P[R_1R_2 \cup R_1R_3 \cup R_2R_3] = 3/10.$$

3.24 a. There are 4 sample points: Boy-Boy, Boy-Girl, Girl-Boy, and Girl-Girl

b. If all the sample points are equally likely, then each would have a probability of ¼.

c. We can estimate the probabilities by using the relative frequency for each sample point. The relative frequency is found by dividing the frequency by the total sample size of 4,208. These estimates are contained in the following table:

Sex Composition of First Two Children	Frequency	Probability
Boy-Boy	1,085	.2578
Boy-Girl	1,086	.2581
Girl-Boy	1,111	.2640
Girl-Girl	926	.2201
TOTAL	4,208	1.0000

d. If having boys or girls "runs in the family", then the probability of getting Boy-Boy or Girl-Girl should be greater than .5. From the above, an estimate of the probability of getting Boy-Boy or Girl-Girl is

$$P(\text{Boy} - \text{Boy or Girl} - \text{Girl}) = \frac{1{,}085 + 926}{4{,}208} = .478$$

Since this is less than .5, there is no evidence that having boys or girls "runs in the family".

3.30 Suppose we label the 4 socks as B_1, B_2, N_1, and N_2. A list of all possible combinations of how the 4 socks could be paired is:

B_1, B_2 and N_1, N_2; B_1, N_1 and B_2, N_2; B_1, N_2 and B_2, N_1

Each of these three combinations are equally likely. Of these 3, only 1 has the socks matched correctly. Thus, the probability of matching the socks is only 1/3 and the probability of mismatching the socks is 2/3.

3.32 a. The simple events of this sample space could be represented by pairs where the first symbol would represent the gene obtained from the mother; the second symbol, the father.

$S = \{BB, Bb, bB, bb\}$

Each of these are equally likely, and the only way a child could have blue eyes would be if "bb" is the child's genetic pair, which has a probability of 1/4.

b. For convenience, let us say that it is the mother whose gene pair is Bb. The only possible simple events here would be:

$S = \{Bb, bb\}$

Again, each of these are equally likely, so that the probability of blue eyes in this case is 1/2.

c. Since the BB parent would donate a "B" gene, the child could not have blue eyes; the probability would be 0.

3.40 a. $P(B^c) = 1 - P(B) = 1 - .7 = .3$

b. $P(A^c) = 1 - P(A) = 1 - .4 = .6$

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .7 - .3 = .8$

3.42 The experiment consists of rolling a pair of fair dice. The simple events are:

1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

Since each die is fair, each simple event is equally likely. The probability of each simple event is 1/36.

a. A: {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

B: {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6)}

$A \cap B$: {(3, 4), (4, 3)}

$A \cup B$: {(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), (4, 1), (4, 2), (4, 3), (4, 5), (4, 6), (1, 6), (2, 5), (5, 2), (6, 1)}

A^c: {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (2, 6), (3, 1), (3, 2), (3, 3), (3, 5), (3, 6), (4, 1), (4, 2), (4, 4), (4, 5), (4, 6), (5, 1), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

(continued)

3.42 (continued)

b. $P(A) = 6\left(\frac{1}{36}\right) = \frac{6}{36} = \frac{1}{6}$

$P(B) = 11\left(\frac{1}{36}\right) = \frac{11}{36}$

$P(A \cap B) = 2\left(\frac{1}{36}\right) = \frac{2}{36} = \frac{1}{18}$

$P(A \cup B) = 15\left(\frac{1}{36}\right) = \frac{15}{36} = \frac{5}{12}$

$P(A^c) = 30\left(\frac{1}{36}\right) = \frac{30}{36} = \frac{5}{6}$

c. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{6} + \frac{11}{36} - \frac{1}{18} = \frac{6+11-2}{36} = \frac{15}{36} = \frac{5}{12}$

d. *A* and *B* are not mutually exclusive. To be mutually exclusive, $P(A \cap B)$ must be 0. Here, $P(A \cap B) = \frac{1}{18}$

3.44 a. $P(A^c) = P(E_3) + P(E_6) = .2 + .3 = .5$

b. $P(B^c) = P(E_1) + P(E_7) = .13 + .06 = .19$

c. $P(A^c \cap B) = P(E_3) + P(E_6) = .2 + .3 = .5$

d. $P(A \cup B) = P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) + P(E_7)$
$= .13 + .05 + .2 + .2 + .06 + .3 + .06 = 1.00$

e. $P(A \cap B) = P(E_2) + P(E_4) + P(E_5) = .05 + .20 + .06 = .31$

f. $P(A^c \cup B^c) = P(E_1) + P(E_7) + P(E_3) + P(E_6) = .13 + .06 + .20 + .30 = .69$

g. No. *A* and *B* are mutually exclusive if $P(A \cap B) = 0$. Here, $P(A \cap B) = .31$.

(3.45) (a) – (g). Solutions were presented during the lecture of Sept. 28.

(h) The possible (unordered) pairs of events are:

AB, AC, AD, BC, BD, CD

The pairs that are mutually exclusive are:

AC, since C ⊂ {off}

BC, since = {off ∩ Low} and

B = {(On ∩ Low) ∪ (On ∩ Med) ∪ {On ∩ High} ∪ {off ∩ Med}}

CD since {off ∩ Low} ⊂ {Low}.

(3.46) Define the following events:

E_1: {3 heads}
E_2: {2 heads}
E_3: {1 heads}
E_4: {0 heads}

a. $A = E_1 \cup E_2 \cup E_3$

$$P(A) = P(E_1) + P(E_2) + P(E_3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$$

b. $A = E_4^c$ $\quad P(A) = 1 - P(E_4) = 1 - \frac{1}{8} = \frac{7}{8}$

(3.48) Define the following event:

A: {Store violates the NIST scanner accuracy standard}

From the problem, we know $P(A) = 52 / 60 = .867$.

The probability that a randomly selected store does not violate the NIST scanner accuracy standard is $P(A^C) = 1 - P(A) = 1 - .867 = .133$.

3.50 a. The student is male and a binge drinker would be the event $B \cap A$.

b. The student is not a binge drinker would be the event A^c.

c. The student is male or lives in a coed dorm is the event $B \cup C$.

d. The student is female and not a binge drinker would be the event $B^c \cap A^c$.

3.52 a. The possible outcomes for this provider are:

(Yes, <50), (Yes, ≥ 50), (No, <50), (No, ≥ 50)

b. We can find reasonable probabilities the 4 sample points by dividing each frequency by the total sample size of 358. The estimates of the probabilities are:

Permit Drug at Home	*Less than 50*	*50 or more*	*Totals*
Yes	.475	.363	.838
No	.134	.028	.162
Totals	.609	.391	1.000

c. Define the following event:

A: {Provider permits home use of abortion drug}
B: {Provider has case load of less than 50 abortions}

$P(A) = .838$

d. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .838 + .609 - .475 = .972$

e. $P(A \cap B) = .475$

3.54 a. *P*(Player is white) = 84 / 368 = .228

b. *P*(Player is a center) = 62 / 368 = .168

c. *P*(Player is African-American and a guard) = 128 / 368 = .348

d. *P*(Player is not a guard) = 1 – *P*(Player is a guard) = 1 – 154 / 368 = 1 – .418 = .582

e. *P*(Player is white or a center)

= *P*(Player is white) + *P*(Player is a center) – *P*(Player is white and a center)

= 84 / 368 + 62 / 368 – 28 / 368 = .228 + .168 - .076 = .320