STAT 213, Section L05

Solutions to Assignment no. 3

3.64 a. $P(A \cap B) = P(A \mid B)P(B) = .6(.2) = .12$

b. $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{.12}{.4} = .3$

3.66 Since A, B, and C are all mutually exclusive, we know that

$P(A \cap B) = P(A \cap C) = P(B \cap C) = 0$

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .30 + .55 - 0 = .85$

b. $P(A \cap B) = 0$

c. $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{.55} = 0$

d. $P(B \cup C) = P(B) + P(C) - P(B \cap C) = .55 + .15 - 0 = .70$

e. No, B and C are not independent events. If B and C are independent events, then

$P(B \mid C) = P(B)$. From the problem, we know P(B) = .55.

$P(B \mid C) = \dfrac{P(B \cap C)}{P(C)} = \dfrac{0}{.15} = 0$. Thus, since $P(B \mid C) \neq P(B)$, events B and C are not independent.

3.70 a. $P(A \cap C) = 0 \Rightarrow A$ and C are mutually exclusive.
$P(B \cap C) = 0 \Rightarrow B$ and C are mutually exclusive.

b. $P(A) = P(1) + P(2) + P(3) = .20 + .05 + .30 = .55$
$P(B) = P(3) + P(4) = .30 + .10 = .40$
$P(C) = P(5) + P(6) = .10 + .25 = .35$
$P(A \cap B) = P(3) = .30$
$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.30}{.40} = .75$

A and B are independent if $P(A \mid B) = P(A)$. Since $P(A \mid B) = .75$ and $P(A) = .55$, A and B are not independent.

Since A and C are mutually exclusive, they are not independent. Similarly, since B and C are mutually exclusive, they are not independent.

(continued)

3.70 (continued)

c. Using the probabilities of simple events,

$$P(A \cup B) = P(1) + P(2) + P(3) + P(4) = .20 + .05 + .30 + .10 = .65$$

Using the additive rule,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = .55 + .40 - .30 = .65$$

Using the probabilities of simple events,

$$P(A \cup C) = P(1) + P(2) + P(3) + P(5) + P(6)$$
$$= .20 + .05 + .30 + .10 + .25 = .90$$

Using the additive rule,

$$P(A \cup C) = P(A) + P(C) - P(A \cap C) = .55 + .35 - 0 = .90$$

3.72 Define the following events:

A: {Child has neuroblastoma}

B: {Child undergoes surgery}

C: {Surgery is successful in curing the disease}

From the problem, we know $P(B \mid A) = .20$ and $P(C \mid B \cap A) = .95$.

We also know that $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)}$ and $P(C \mid B \cap A) = \dfrac{P(C \cap B \cap A)}{P(B \cap A)}$

Thus, $P(C \cap B \mid A) = \dfrac{P(C \cap B \cap A)}{P(A)} = \dfrac{P(C \cap B \cap A)}{P(B \cap A)} \dfrac{P(B \cap A)}{P(A)} = P(C \mid B \cap A)P(B|A)$

$= .95(.20) = .19$

3.80 Define the following events:

F: {Fight}
N: {No fight}
W: {Initiator Wins}
T: {No Clear Winner}
L: {Initiator Loses}

a. $P(W|F) = \dfrac{P(W \cap F)}{P(F)} = \dfrac{26/167}{64/167} = \dfrac{26}{64} = .406$

b. $P(W|N) = \dfrac{P(W \cap N)}{P(N)} = \dfrac{80/167}{103/167} = \dfrac{80}{103} = .777$

c. Two events are independent if $P(W \mid N) = P(W)$.
$P(W) = \dfrac{106}{167} = .635$ which does not equal $P(W|N) = .777$. Thus, "no fight" and "initiator wins" are not independent.

3.84 Define the following events:

A: {Man never smoked cigars}

B: {Man formerly smoked cigars}

C: {Man currently smokes cigars}

D: {Man died from Cancer}

a. $P(A \cap D) = \dfrac{782}{137{,}243} = .006$

b. $P(B \cap D) = \dfrac{91}{137{,}243} = .0007$

c. $P(C \cap D) = \dfrac{141}{137{,}243} = .001$

d. $P(D \mid C) = \dfrac{P(D \cap C)}{P(C)} = \dfrac{141/137{,}243}{7{,}866/137{,}243} = \dfrac{141}{7{,}866} = .018$

e. $P(D \mid A) = \dfrac{P(D \cap A)}{P(A)} = \dfrac{782/137{,}243}{121{,}529/137{,}243} = \dfrac{782}{121{,}529} = .006$

3.132 First, we find the following probabilities:

$$P(A \cap B_1) = P(A \mid B_1)P(B_1) = .4(.2) = .08$$
$$P(A \cap B_2) = P(A \mid B_2)P(B_2) = .25(.15) = .0375$$
$$P(A \cap B_3) = P(A \mid B_3)P(B_3) = .6(.65) = .5075$$
$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) = .08 + .0375 + .39 = .5075$$

a. $P(B_1 \mid A) = \dfrac{P(A \cap B_1)}{P(A)} = \dfrac{.08}{.5075} = .158$

b. $P(B_2 \mid A) = \dfrac{P(A \cap B_2)}{P(A)} = \dfrac{.0375}{.5075} = .074$

c. $P(B_3 \mid A) = \dfrac{P(A \cap B_3)}{P(A)} = \dfrac{.39}{.5075} = .768$

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3.134 a. $$P(G_1 \mid D) = \frac{P(G_1 \cap D)}{P(D)} = \frac{P(D \mid G_1)P(G_1)}{P(D)} = \frac{.5(.2)}{.34} = .294$$

b. $$P(G_2 \mid D) = \frac{P(G_2 \cap D)}{P(D)} = \frac{P(D \mid G_2)P(G_2)}{P(D)} = \frac{.3(.8)}{.34} = .706$$

3.136 a. Converting the percentages to probabilities,

$P(275 - 300) = .52$, $P(305 - 325) = .39$, and $P(330 - 350) = .09$.

b. Using Bayes Theorem,

$$P(275-300 \mid CC) = \frac{P(275-300 \cap CC)}{P(CC)}$$

$$= \frac{P(CC \mid 275-300)P(275-300)}{P(CC \mid 275-300)P(275-300) + P(CC \mid 305-325)P(305-325) + P(CC \mid 330-350)P(330-350)}$$

$$= \frac{.775(.52)}{.775(.52) + .77(.39) + .86(.09)} = \frac{.403}{.403 + .3003 + .0774} = \frac{.403}{.7807} = .516$$

3.140 Define the following events:

S: {System shuts down}
F_1: {Hardware failure}
F_2: {Software failure}
F_3: {Power failure}

From the Exercise, we know:

$P(F_1) = .01$, $P(F_2) = .05$, and $P(F_3) = .02$. Also, $P(S|F_1) = .73$, $P(S|F_2) = .12$, and $P(S|F_3) = .88$.

The probability that the current shutdown is due to a hardware failure is:

$$P(F_1 \mid S) = \frac{P(F_1 \cap S)}{P(S)} = \frac{P(S \mid F_1)P(F_1)}{P(S \mid F_1)P(F_1) + P(S \mid F_2)P(F_2) + P(S \mid F_3)P(F_3)}$$
$$= \frac{.73(.01)}{.73(.01) + .12(.05) + .88(.02)} = \frac{.0073}{.0073 + .006 + .0176} = \frac{.0073}{.0309} = .2362$$

The probability that the current shutdown is due to a software failure is:

$$P(F_2 \mid S) = \frac{P(F_2 \cap S)}{P(S)} = \frac{P(S \mid F_2)P(F_2)}{P(S \mid F_1)P(F_1) + P(S \mid F_2)P(F_2) + P(S \mid F_3)P(F_3)}$$
$$= \frac{.12(.05)}{.73(.01) + .12(.05) + .88(.02)} = \frac{.006}{.0073 + .006 + .0176} = \frac{.006}{.0309} = .1942$$

The probability that the current shutdown is due to a power failure is:

$$P(F_3 \mid S) = \frac{P(F_3 \cap S)}{P(S)} = \frac{P(S \mid F_3)P(F_3)}{P(S \mid F_1)P(F_1) + P(S \mid F_2)P(F_2) + P(S \mid F_3)P(F_3)}$$
$$= \frac{.88(.02)}{.73(.01) + .12(.05) + .88(.02)} = \frac{.0176}{.0073 + .006 + .0176} = \frac{.0176}{.0309} = .5696$$

4.14 a. x may take on the values −4, 0, 1, or 3.

b. The value 1 is more likely than any of the other three since its probability of .4 is the maximum probability of the probability distribution.

c. $P(x > 0) = P(x = 1) + P(x = 3) = .4 + .3 = .7$

d. $P(x = -2) = 0$

4.16 a. This is a valid distribution because $\sum p(x) = 1$ and $p(x) \geq 0$ for all values of x.

b. This is *not* a valid distribution because $\sum p(x) = .95 \neq 1$.

c. This is *not* a valid distribution because one of the probabilities is negative.

d. The sum of the probabilities over all possible values of the random variable is greater than 1, so this is *not* a valid probability distribution.

4.18 a. $P(x \leq 0) = P(x = -2) + P(x = -1) + P(x = 0) = .10 + .15 + .40 = .65$

b. $P(x > -1) = P(x = 0) + P(x = 1) + P(x = 2) = .40 + .30 + .05 = .75$

c. $P(-1 \leq x \leq 1) = P(x = -1) + P(x = 0) + P(x = 1) = .15 + .40 + .30 = .85$

d. $P(x < 2) = 1 - P(x = 2) = 1 - .05 = .95$

e. $P(-1 < x < 2) = P(x = 0) + P(x = 1) = .40 + .30 = .70$

f. $P(x < 1) = P(x = -2) + P(x = -1) + P(x = 0) = .10 + .15 + .40 = .65$

4.26 a. $p(1) = (.23)(.77)^{1-1} = .23(.77)^0 = .23$. The probability that a contaminated cartridge is selected on the first sample is .23.

b. $p(5) = (.23)(.77)^{5-1} = .23(.77)^4 = .081$. The probability that the first contaminated cartridge will be selected on the 5^{th} sample is .081.

c. $P(x \geq 2) = 1 - P(x \leq 1) = 1 - p(1) = 1 - .23 = .77$. The probability that the first contaminated cartridge will be selected on the second trial or later is .77.

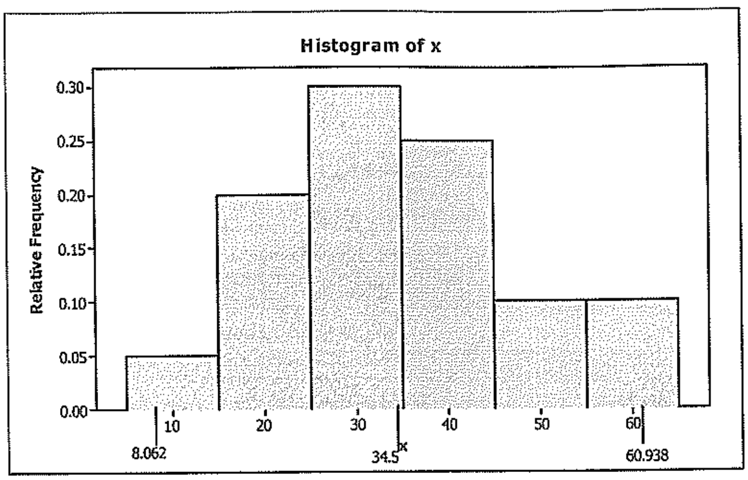
4.28 a. $P(x \geq 10) = p(10) + p(15) + p(30) + p(50) + p(99) = .05 + .085 + .01 + .004 + .001 = .150$

b. $P(x < 0) = p(-4) + p(-2) + p(-1) = .02 + .06 + .07 = .15$

4.36 a. $\mu = E(x) = \sum xp(x)$
$$= 10(.05) + 20(.20) + 30(.30) + 40(.25) + 50(.10) + 60(.10)$$
$$= .5 + 4 + 9 + 10 + 5 + 6 = 34.5$$

$$\sigma^2 = E(x-\mu)^2 = \sum (x-\mu)^2 p(x)$$
$$= (10-34.5)^2(.05) + (20-34.5)^2(.20) + (30-34.5)^2(.30)$$
$$+ (40-34.5)^2(.25) + (50-34.5)^2(.10) + (60-34.5)^2(.10)$$
$$= 30.0125 + 42.05 + 6.075 + 7.5625 + 24.025 + 65.025 = 174.75$$
$$\sigma = \sqrt{174.75} = 13.219$$

b.



c. $\mu \pm 2\sigma \Rightarrow 34.5 \pm 2(13.219) \Rightarrow 34.5 \pm 26.438 \Rightarrow (8.062, 60.938)$

$$P(8.062 < x < 60.938) = p(10) + p(20) + p(30) + p(40) + p(50) + p(60)$$
$$= .05 + .20 + .30 + .25 + .10 + .10 = 1.00$$

4.38 a. $\mu = E(x) = \sum xp(x) = -4(.02) + (-3)(.07) + (-2)(.10) + (-1)(.15) + 0(.3)$
$$+ 1(.18) + 2(.10) + 3(.06) + 4(.02)$$
$$= -.08 - .21 - .2 - .15 + 0 + .18 + .2 + .18 + .08 = 0$$

$$\sigma^2 = E[(x-\mu)^2] = \sum (x-\mu)^2 p(x)$$
$$= (-4-0)^2(.02) + (-3-0)^2(.07) + (-2-0)^2(.10)$$
$$+ (-1-0)^2(.15) + (0-0)^2(.30) + (1-0)^2(.18)$$
$$+ (2-0)^2(.10) + (3-0)^2(.06) + (4-0)^2(.02)$$
$$= .32 + .63 + .4 + .15 + 0 + .18 + .4 + .54 + .32 = 2.94$$
$$\sigma = \sqrt{2.94} = 1.715$$

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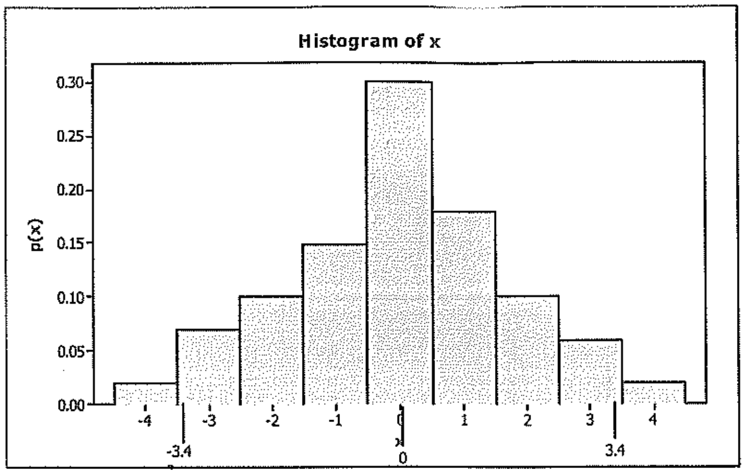
4.38 (continued)

b.



$\mu \pm 2\sigma \Rightarrow 0 \pm 2(1.715) \Rightarrow 0 \pm 3.430 \Rightarrow (-3.430, 3.430)$

c. $P(-3.430 < x < 3.430) = p(-3) + p(-2) + p(-1) + p(0) + p(1) + p(2) + p(3)$
$= .07 + .10 + .15 + .30 + .18 + .10 + .06 = .96$

4.42 $\mu = E(x) = \sum xp(x) = 1(.40) + 2(.54) + 3(.02) + 4(.04) = .40 + 1.08 + .06 + .16 = 1.70$

The average number of insect eggs on a blade of water hyacinth is 1.70.

4.44 Let x = winnings in the Florida lottery. The probability distribution for x is:

x	$p(x)$
−\$1	13,999,999/14,000,000
\$6,999,999	1/14,000,000

The expected net winnings would be:

$\mu = E(x) = (-1)(13{,}999{,}999/14{,}000{,}000) + 6{,}999{,}999(1/14{,}000{,}000) = -\$.50$

The average winnings of all those who play the lottery is −\$.50.

4.46 a. Since there are 20 possible outcomes that are all equally likely, the probability of any of the 20 numbers is 1/20. The probability distribution of x is:

$P(x = 5) = 1/20 = .05$; $P(x = 10) = 1/20 = .05$; etc.

x	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
$p(x)$	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05	.05

b. $E(x) = \sum xp(x) = 5(.05) + 10(.05) + 15(.05) + 20(.05) + 25(.05) + 30(.05) + 35(.05)$
$+ 40(.05) + 45(.05) + 50(.05) + 55(.05) + 60(.05) + 65(.05) + 70(.05) + 75(.05)$
$+ 80(.05) + 85(.05) + 90(.05) + 95(.05) + 100(.05) = 52.5$

c. $\sigma^2 = E(x - \mu)^2 = \sum (x-\mu)^2 p(x) = (5 - 52.5)^2(.05) + (10 - 52.5)^2(.05)$
$+ (15 - 52.5)^2(.05) + (20 - 52.5)^2(.05) + (25 - 52.5)^2(.05) + (30 - 52.5)^2(.05)$
$+ (35 - 52.5)^2(.05) + (40 - 52.5)^2(.05) + (45 - 52.5)^2(.05) + (50 - 52.5)^2(.05)$
$+ (55 - 52.5)^2(.05) + (60 - 52.5)^2(.05) + (65 - 52.5)^2(.05) + (70 - 52.5)^2(.05)$
$+ (75 - 52.5)^2(.05) + (80 - 52.5)^2(.05) + (85 - 52.5)^2(.05) + (90 - 52.5)^2(.05)$
$+ (95 - 52.5)^2(.05) + (100 - 52.5)^2(.05)$
$= 831.25$

$\sigma = \sqrt{\sigma^2} = \sqrt{831.25} = 28.83$

Since the uniform distribution is not mound-shaped, we will use Chebyshev's theorem to describe the data. We know that at least 8/9 of the observations will fall with 3 standard deviations of the mean and at least 3/4 of the observations will fall within 2 standard deviations of the mean. For this problem,

$\mu \pm 2\sigma \Rightarrow 52.5 \pm 2(28.83) \Rightarrow 52.5 \pm 57.66 \Rightarrow (-5.16, 110.16)$. Thus, at least 3/4 of the data will fall between −5.16 and 110.16. For our problem, all of the observations will fall within 2 standard deviations of the mean. Thus, x is just as likely to fall within any interval of equal length.