STAT 213 L05

Solutions to Assignment #4

4.54 a. $p(0) = \binom{3}{0}(.3)^0(.7)^{3-0} = \frac{3!}{0!3!}(.3)^0(.7)^3 = \frac{3\cdot 2\cdot 1}{1\cdot 3\cdot 2\cdot 1}(1)(.7)^3 = .343$

$p(1) = \binom{3}{1}(.3)^1(.7)^{3-1} = \frac{3!}{1!2!}(.3)^1(.7)^2 = .441$

$p(2) = \binom{3}{2}(.3)^2(.7)^{3-2} = \frac{5!}{2!1!}(.3)^2(.7)^1 = .189$

$p(3) = \binom{3}{3}(.3)^3(.7)^{3-3} = \frac{5!}{3!0!}(.3)^3(.7)^0 = .027$

b.

x	$p(x)$
0	.343
1	.441
2	.189
3	.027

4.56 a. $P(x = 2) = P(x \le 2) - P(x \le 1) = .167 - .046 = .121$ (from Table II, Appendix A)

b. $P(x \le 5) = .034$

c. $P(x > 1) = 1 - P(x \le 1) = 1 - .919 = .081$

4.58 a. The simple events listed below are all equally likely, implying a probability of 1/32 for each. The list is in a regular pattern such that the first simple event would yield $x = 0$, the next five yield $x = 1$, the next ten yield $x = 2$, the next ten also yield $x = 3$, the next five yield $x = 4$, and the final one yields $x = 5$. The resulting probability distribution is given below the simple events.

$$\begin{bmatrix} \text{FFFFF, FFFFS, FFFSF, FFSFF, FSFFF, SFFFF, FFFSS, FFSFS} \\ \text{FSFFS, SFFFS, FFSSF, FSFSF, SFFSF, FSSFF, SFSFF, SSFFF} \\ \text{FFSSS, FSFSS, SFFSS, FSSFS, SFSFS, SSFFS, FSSSF, SFSSF} \\ \text{SSFSF, SSSFF, FSSSS, SFSSS, SSFSS, SSSFS, SSSSF, SSSSS} \end{bmatrix}$$

x	0	1	2	3	4	5
p(x)	1/32	5/32	10/32	10/32	5/32	1/32

b. $$P(x=0) = \frac{5!}{0!5!}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(1)(5\cdot4\cdot3\cdot2\cdot1)}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5$$
$$= 1\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5 = \frac{1}{32} = .03125$$

$$P(x=1) = \frac{5!}{1!4!}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(1)(4\cdot3\cdot2\cdot1)}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4$$
$$= 5\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^4 = \frac{5}{32} = .15625$$

$$P(x=2) = \frac{5!}{2!3!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(2\cdot1)(3\cdot2\cdot1)}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3$$
$$= 10\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = \frac{10}{32} = .3125$$

$$P(x=3) = \frac{5!}{3!2!}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(3\cdot2\cdot1)(2\cdot1)}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$$
$$= 10\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = \frac{10}{32} = .3125$$

$$P(x=4) = \frac{5!}{4!1!}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(4\cdot3\cdot2\cdot1)(1)}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1$$
$$= 5\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5}{32} = .15625$$

(continued)

4.59 (b) (continued)

$$P(x=5) = \frac{5!}{5!0!}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 = \frac{5\cdot4\cdot3\cdot2\cdot1}{(5\cdot4\cdot3\cdot2\cdot1)(1)}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$$
$$= 1\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0 = \frac{1}{32} = .15625$$

c. From Table II, $n = 5, p = .5$:

$P(x=0) = .031$
$P(x=1) = .188 - .031 = .157$
$P(x=2) = .500 - .188 = .312$
$P(x=3) = .812 - .500 = .312$
$P(x=4) = .969 - .812 = .157$
$P(x=5) = 1 - .969 = .031$

4.60 a. We will check the characteristics of a binomial random variable:

1. This experiment consists of $n = 5$ identical trials.

2. There are only 2 possible outcomes for each trial. A brand of bottled water can use tap water (S) or not (F).

3. The probability of S remains the same from trial to trial. In this case, $p = P(S) \approx .25$ for each trial.

4. The trials are independent. Since there are a finite number of brands of bottled water, the trials are not exactly independent. However, since the number of brands of bottled water is large compared to the sample size of 5, the trials are close enough to being independent.

5. x = number of brands of bottled water using tap water in 5 trials.

b. The formula for finding the binomial probabilities is:

$$p(x) = \binom{5}{x}.25^x(.75)^{5-x} \quad \text{for } x = 0, 1, 2, 3, 4, 5$$

c. $$P(x=2) = p(2) = \binom{5}{2}.25^2(.75)^{5-2} = \frac{5!}{2!3!}.25^2(.75)^3$$
$$= \frac{5\cdot4\cdot3\cdot2\cdot1}{2\cdot1\cdot3\cdot2\cdot1}(.0625)(.421875) = .2637$$

d. $$P(x\le1) = p(0)+p(1) = \binom{5}{0}.25^0(.75)^{5-0} + \binom{5}{1}.25^1(.75)^{5-1}$$
$$= \frac{5!}{0!5!}.25^0(.75)^5 + \frac{5!}{1!4!}.25^1(.75)^4$$
$$= .75^5 + 5(.25)(.75)^4 = .2373 + .3955 = .6328$$

4.62 a. Let x = number of births in 1,000 that take place by Caesarian section.
$E(x) = np = 1000(.29) = 290$.

b. $\sigma = \sqrt{npq} = \sqrt{1000(.29)(.71)} = 14.3492 = 13.0996$

4.64 a. From the problem, x is a binomial random variable with $n = 3$ and $p = .6$.

$$P(x=0) = \binom{3}{0} .6^0 .4^{3-0} = \frac{3!}{0!3!} .6^0 .4^3 = .064.$$

b. $P(x \geq 1) = 1 - P(x = 0) = 1 - .064 = .936.$

c. $\mu = E(x) = np = 3(.6) = 1.8$

$\sigma = \sqrt{npq} = \sqrt{3(.6)(.4)} = .8485$

In samples of 3 parents, on the average, 1.8 condone spanking.

4.70 Let $n = 20$, $p = .5$, and x = number of correct questions in 20 trials. Then x has a binomial distribution. We want to find k such that:

$P(x \geq k) < .05$
or $1 - P(x \leq k - 1) < .05 \Rightarrow P(x \leq k - 1) > .95$
$\Rightarrow k - 1 = 14 \Rightarrow k = 15$
(from Table II, Appendix A)

Note: $P(x \geq 14) = 1 - P(x \leq 13) = 1 - .942 = .058$
$P(x \geq 15) = 1 - P(x \leq 14) = 1 - .979 = .021$

Thus, to have the probability less than .05, the lowest passing grade should be 15.

4.72 Let x = Number of boys in 24 children. Then x is a binomial random variable with $n = 24$ and $p = .5$.

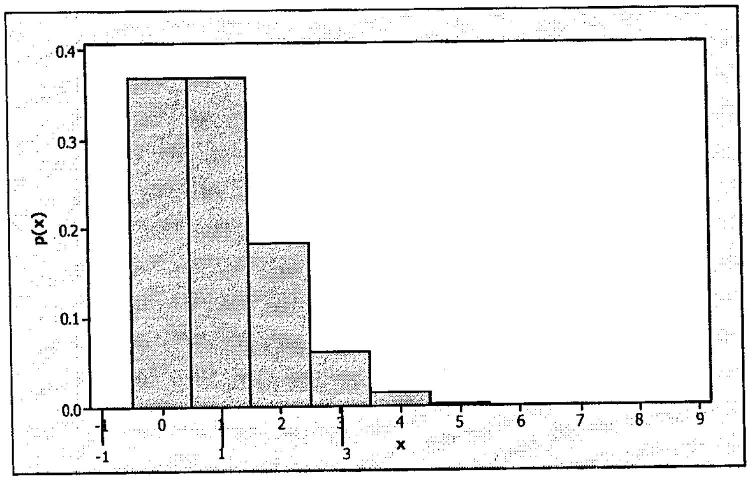
$\mu = E(x) = np = 24(.5) = 12$

$\sigma = \sqrt{npq} = \sqrt{24(.5)(.5)} = \sqrt{6} = 2.4495$

A value of 21 boys out of 24 children would have a z-score of $z = \frac{21-12}{2.4495} = 3.67$. A value that is 3.67 standard deviations above the mean would be highly unlikely. Thus, we would agree with the statement, "Rodgers men produce boys."

4.80 a. For $\lambda = 1$, from Table III, Appendix A.

$$p(0) = P(x \le 0) = .368$$
$$p(1) = P(x \le 1) - P(x \le 0) = .736 - .368 = .368$$
$$p(2) = P(x \le 2) - P(x \le 1) = .920 - .736 = .184$$
$$p(3) = P(x \le 3) - P(x \le 2) = .981 - .920 = .061$$
$$p(4) = P(x \le 4) - P(x \le 3) = .996 - .981 = .015$$
$$p(5) = P(x \le 5) - P(x \le 4) = .999 - .996 = .003$$
$$p(6) = P(x \le 6) - P(x \le 5) = 1.000 - .999 = .001$$
$$p(7) = P(x \le 7) - P(x \le 6) = 1.000 - 1.000 = 0$$
$$p(8) = P(x \le 8) - P(x \le 7) = 1.000 - 1.000 = 0$$
$$p(9) = P(x \le 9) - P(x \le 8) = 1.000 - 1.000 = 0$$



b. $\mu = \lambda = 1,\ \sigma^2 = \lambda = 1,\ \sigma = \sqrt{1} = 1$

$\mu \pm 2\sigma \Rightarrow 1 \pm 2(1) \Rightarrow 1 \pm 2 \Rightarrow (-1, 3)$

c. $P(-1 < x < 3) = p(0) + p(1) + p(2) + p(3) = .368 + .368 + .184 = .920$

4.82 From Table II, Appendix A, with $n = 25$ and $p = .05$,

$$p(0) = P(x \le 0) = .277$$
$$p(1) = P(x \le 1) - P(x \le 0) = .642 - .277 = .365$$
$$p(2) = P(x \le 2) - P(x \le 1) = .873 - .642 = .231$$

$$\lambda = \mu = np = 25(.05) = 1.25$$

$$p(0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{1.25^0 e^{-1.25}}{0!} = e^{-1.25} = .287$$

$$p(1) = \frac{1.25^1 e^{-1.25}}{1!} = 1.25e^{-1.25} = .358$$

$$p(2) = \frac{1.25^2 e^{-1.25}}{2!} = \frac{1.5625e^{-1.25}}{2} = .224$$

Note that these probabilities are very close.

4.84 Let x = number of extrasolar planet transits for 10,000 stars. Then x has a Poisson distribution with $\lambda = 5.6$.

$$P(x > 10) = 1 - P(x \le 10) = 1 - .972 = .028$$

4.90 a. $P(x \le 3) = .010$ using Table III, Appendix A with $\lambda = 10$.

b. Yes. The probability of observing 3 or fewer crimes in a year if the mean is still 10 is extremely small. This is evidence that the Crime Watch group has been effective in this neighborhood.

4.104 a. Let x = number of defective items in a sample of size 4. For this problem, x is a hypergeometric random variable with $N = 10$, $n = 4$, and $r = 1$. You will accept the lot if you observe no defectives.

$$P(x=0) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{1}{0}\binom{10-1}{4-0}}{\binom{10}{4}} = \frac{\frac{1!}{0!1!}\frac{9!}{4!5!}}{\frac{10!}{4!6!}} = \frac{1(84)}{210} = .4$$

b. If $r = 2$,

$$P(x=0) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{2}{0}\binom{10-2}{4-0}}{\binom{10}{4}} = \frac{\frac{2!}{0!2!}\frac{8!}{4!4!}}{\frac{10!}{4!6!}} = \frac{1(70)}{210} = .333$$

4.106 Let x = number of British bird species sampled that inhabit a butterfly hotspot in 4 trials. Because the sampling is done without replacement, x is a hypergeometric random variable with $N = 10$, $n = 4$, and $r = 7$.

a. $$P(x=2) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{7}{2}\binom{10-7}{4-2}}{\binom{10}{4}} = \frac{\frac{7!}{2!5!}\frac{3!}{2!1!}}{\frac{10!}{4!6!}} = \frac{21(3)}{210} = .3$$

b. $P(x \ge 1) = 1$ because the only values x can take on are 1, 2, 3, or 4.

4.122 Let x = number of times the vehicle is used in a day. Then x has a Poisson distribution with $\lambda = 1.3$.

a. $P(x = 2) = P(x \le 2) - P(x \le 1) = .857 - .627 = .230$ (from Table III, Appendix A)

b. $P(x > 2) = 1 - P(x \le 2) = 1 - .857 = .143$

c. $P(x = 3) = P(x \le 3) - P(x \le 2) = .957 - .857 = .100$

4.124 a. Let x = number of trees infected with the Dutch elm disease in the two trees purchased. For this problem, x is a hypergeometric random variable with $N = 10$, $n = 2$, and $r = 3$.

The probability that both trees will be healthy is:

$$P(x=0) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{3}{0}\binom{10-3}{2-0}}{\binom{10}{2}} = \frac{\frac{3!}{0!3!}\frac{7!}{2!5!}}{\frac{10!}{2!8!}} = \frac{1(21)}{45} = .467$$

b. The probability that at least one tree will be infected is:

$P(x \ge 1) = 1 - P(x = 0) = 1 - .467 = .533$.

4.128 Let x = number of defective CD mini-rack systems in 5 trials. Since the selection is done without replacement, x is a hypergeometric random variable with $N = 10$, $n = 5$, and $r = 3$.

a. The probability that the shipment will be rejected is the same as the probability that at least one defective CD mini-rack system is selected:

$$P(x \ge 1) = 1 - P(x=0) = 1 - \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = 1 - \frac{\binom{3}{0}\binom{10-3}{5-0}}{\binom{10}{5}}$$

$$= 1 - \frac{\frac{3!}{0!3!}\frac{7!}{5!2!}}{\frac{10!}{5!5!}} = 1 - \frac{1(21)}{252} = 1 - .083 = .917$$

b. If 6 CD mini-rack systems are selected, the shipment will be accepted if none of the systems are defective:

$$P(x=0) = \frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}} = \frac{\binom{3}{0}\binom{10-3}{6-0}}{\binom{10}{6}} = \frac{\frac{3!}{0!3!}\frac{7!}{6!1!}}{\frac{10!}{6!4!}} = \frac{1(7)}{210} = .033$$