STAT 213 L05

Solutions to Assignment #5

(5.5) (a) $f(x) = \frac{1}{2}$, $2 \le x \le 4$
$= 0$, elsewhere

(b) $EX = \frac{a+b}{2} = \frac{2+4}{2} = \underline{\underline{3}}$

$\sigma(x) = \frac{b-a}{\sqrt{12}} = \frac{4-2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \underline{\underline{0.5774}}$

(c) $P[\mu - \sigma \le X \le \mu + \sigma] = P[3 - .577 \le X \le 3 + .577]$
$= \frac{1}{2}[3.577 - 2.423] = \frac{1}{2}\, 2(.577) = \underline{\underline{.577}}$

(d) $P[X > 2.78] = P[2.78 < X < 4] = \frac{1}{2}(4 - 2.78) = \underline{\underline{0.61}}$

(e) $P[2.4 \le X \le 3.7] = \frac{1}{2}(3.7 - 2.4) = \underline{\underline{0.65}}$

(f) $P[X < 2] = \int_{-\infty}^{0} 0\, dx = \underline{\underline{0}}$.

(5.6) From Exercise 5.5, $f(x) = \frac{1}{2}$ $(2 \le x \le 4)$

a. $P(x \ge a) = .5 \Rightarrow (4 - a)\left(\frac{1}{2}\right) = .5$
$\Rightarrow 4 - a = 1$
$\Rightarrow a = 3$

b. $P(x \le a) = .2 \Rightarrow (a - 2)\left(\frac{1}{2}\right) = .2$
$\Rightarrow a - 2 = .4$
$\Rightarrow a = 2.4$

(continued)

5.6 (continued)

c. $P(x \le a) = 0 \Rightarrow (a-2)\left(\frac{1}{2}\right) = 0$

$\Rightarrow \quad a - 2 = 0$

$\Rightarrow \quad a = 2$ or any number less than 2

d. $P(2.5 \le x \le a) = .5 \Rightarrow (a-2.5)\left(\frac{1}{2}\right) = .5$

$\Rightarrow \quad a - 2.5 = 1$

$\Rightarrow \quad a = 3.5$

5.8 $\mu = \frac{c+d}{2} = 50 \Rightarrow c + d = 100 \Rightarrow c = 100 - d$

$\sigma = \frac{d-c}{\sqrt{12}} = 5 \Rightarrow d - c = 5\sqrt{12}$

Substituting, $d - (100 - d) = 5\sqrt{12} \Rightarrow 2d - 100 = 5\sqrt{12}$

$\Rightarrow 2d = 100 + 5\sqrt{12}$

$\Rightarrow d = \frac{100 + 5\sqrt{12}}{2}$

$\Rightarrow d = 58.66$

Since $c + d = 100 \Rightarrow c + 58.66 = 100$

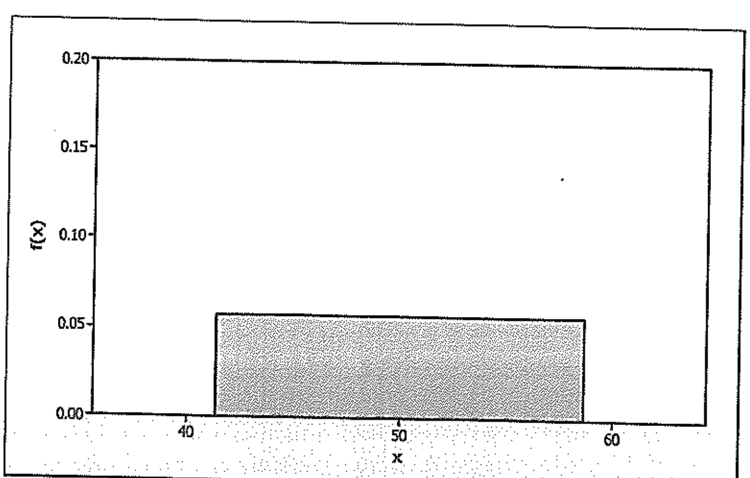
$\Rightarrow c = 41.34$

$f(x) = \frac{1}{d-c} \qquad (c \le x \le d)$

$\frac{1}{d-c} = \frac{1}{58.66 - 41.34} = \frac{1}{17.32} = .058$

Therefore, $f(x) = \begin{cases} .058 & 41.34 \le x \le 58.66 \\ 0 & \text{otherwise} \end{cases}$

The graph of the probability distribution for x is given here.



5.10 a. For this problem, $c = 0$ and $d = 1$.

$$f(x) = \begin{cases} \frac{1}{d-c} = \frac{1}{1-0} & (0 \le x \le 1) \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{c+d}{2} = \frac{0+1}{2} = .5$$

$$\sigma^2 = \frac{(d-c)^2}{12} = \frac{(1-0)^2}{12} = \frac{1}{12} = .0833$$

b. $P(.2 < x < .4) = (.4 - .2)(1) = .2$

c. $P(x > .995) = (1 - .995)(1) = .005$. Since the probability of observing a trajectory greater than .995 is so small, we would not expect to see a trajectory exceeding .995.

5.16 Let x = length of time a bus is late. Then x is a uniform random variable with probability distribution:

$$f(x) = \begin{cases} \frac{1}{20} & (0 \le x \le 20) \\ 0 & \text{otherwise} \end{cases}$$

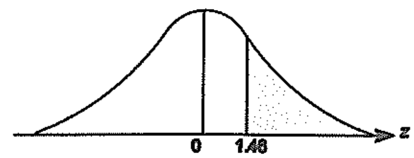
a. $\mu = \frac{0+20}{2} = 10$

b. $P(x \ge 19) = (20 - 19) \cdot \left(\frac{1}{20}\right) = \frac{1}{20} = .05$

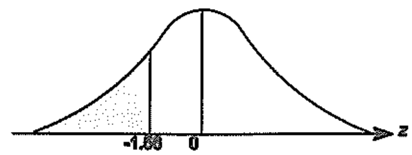
c. It would be doubtful that the director's claim is true, since the probability of the being more than 19 minutes late is so small.

5.24 Using Table IV, Appendix A:

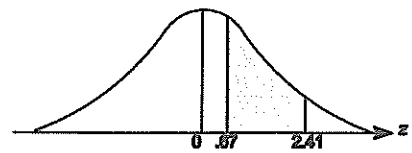
a. $P(z > 1.46) = .5 \;\; P(0 < z \leq 1.46)$
$= .5 - .4279 = .0721$



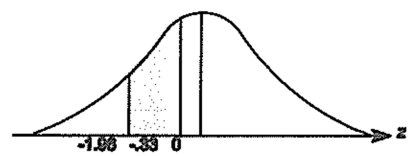
b. $P(x < -1.56) = .5 - P(-1.56 \leq z < 0)$
$= .5 - .4406 = .0594$



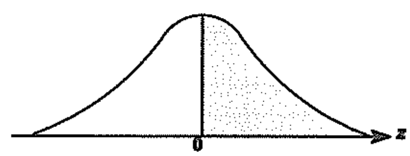
c. $P(.67 \leq z \leq 2.41)$
$= P(0 < z \leq 2.41) - P(0 < z < .67)$
$= .4920 - .2486 = .2434$



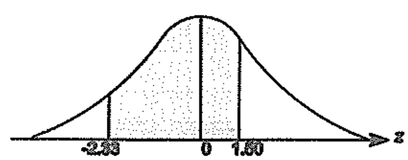
d. $P(-1.96 \leq z < -.33)$
$= P(-1.96 \leq z < 0) - P(-.33 \leq z < 0)$
$= .4750 - .1293 = .3457$



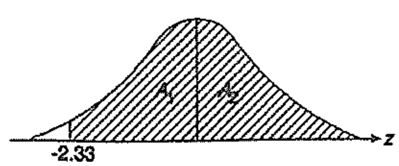
e. $P(z \geq 0) = .5$



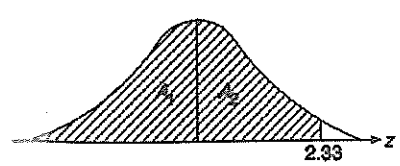
f. $P(-2.33 < z < 1.50)$
$= P(-2.33 < z < 0) + P(0 < z < 1.50)$
$= .4901 + .4332 = .9233$



g. $P(z \geq -2.33) = P(-2.33 \leq \; z \leq 0) + P(z \geq 0)$
$= .4901 + .5000$
$= .9901$

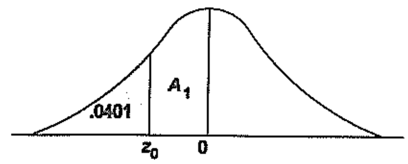


h. $P(z < 2.33) = P(z \leq 0) + P(0 \leq z \leq 2.33)$
$= .5000 + .4901$
$= .9901$



5.28 Using Table IV of Appendix A:

a. $P(z \le z_0) = .0401$

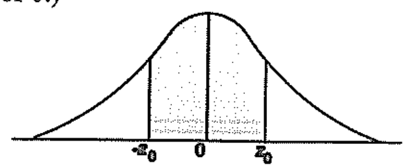


$A_1 = .5000 - .0401 = .4591$

Look up the area .4591 in the body of Table IV;
$z_0 = -1.75$

(z_0 is negative since the graph shows z_0 is on the left side of 0.)

b. $P(-z_0 \le z \le z_0) = .95$

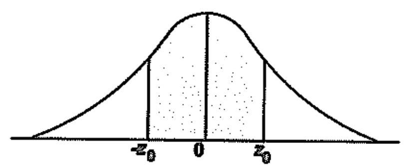


$P(-z_0 \le z \le z_0) = 2P(0 \le z \le z_0)$
$2P(0 \le z \le z_0) = .95$

Therefore, $P(0 \le z \le z_0) = .4750$

Look up the area .4750 in the body of Table IV; $z_0 = 1.96$

c. $P(-z_0 \le z < z_0) = .90$

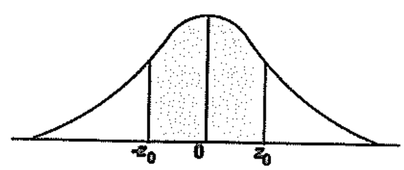


$P(-z_0 \le z < z_0) = 2P(0 \le z < z_0)$
$2P(0 \le z < z_0) = .90$

Therefore, $P(0 \le z \le z_0) = .45$

Look up the area .45 in the body of Table IV; $z_0 = 1.645$ (.45 is half way between .4495 and .4505; therefore, we average the z-scores $\frac{1.64 + 1.65}{2} = 1.645$)

d. $P(-z_0 \le z \le z_0) = .8740$

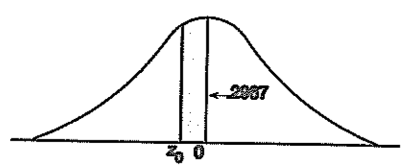


$P(-z_0 \le z \le z_0) = 2P(0 \le z \le z_0)$
$2P(0 \le z \le z_0) = .8740$

Therefore, $P(0 \le z \le z_0) = .4370$

Look up the area .4370 in the body of Table IV; $z_0 = 1.53$

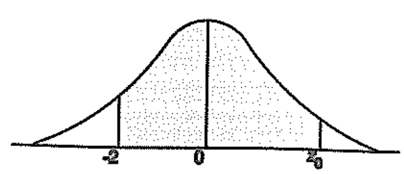
e. $P(z_0 \le z \le 0) = .2967$



$P(z_0 \le z \le 0) = P(0 \le z \le -z_0)$

Look up the area .2967 in the body of Table IV; $z_0 = -.83$

f. $P(-2 < z < z_0) = .9710$



$P(-2 < z < z_0)$
$= P(-2 < z < 0) + P(0 < z < z_0) = .9710$

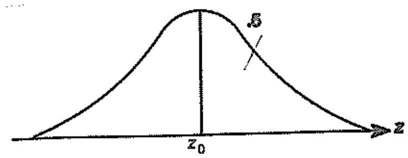
$P(0 < z < 2) + P(0 < z < z_0) = .9710$

Thus, $P(0 < z < z_0) = .9710 - P(0 < z < 2) = .9710 - .4772 = .4938$

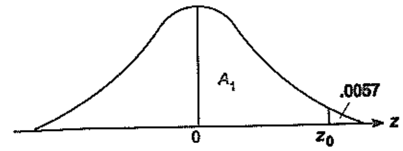
Look up the area .4938 in the body of Table IV; $z_0 = 2.50$

(continued)

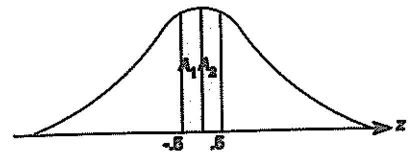
g. $P(z \geq z_0) = .5$
$z_0 = 0$



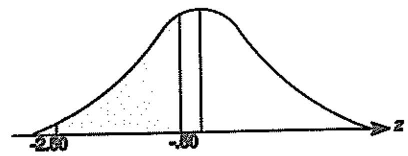
h. $P(z \geq z_0) = .0057$
$A_1 = .5 - .0057 = .4943$
Looking up the area .4943 in Table IV gives $z_0 = 2.53$.



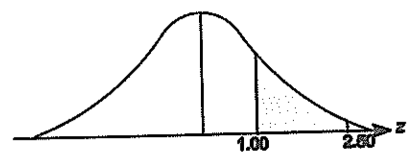
5.30 a. $P(10 \leq x \leq 12) = P\left(\frac{10-11}{2} \leq z \leq \frac{12-11}{2}\right)$
$= P(-0.50 \leq z \leq 0.50)$
$= A_1 + A_2$
$= .1915 + .1915 = .3830$



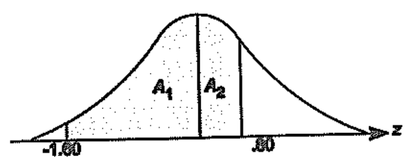
b. $P(6 \leq x \leq 10) = P\left(\frac{6-11}{2} \leq z \leq \frac{10-11}{2}\right)$
$= P(-2.50 \leq z \leq -0.50)$
$= P(-2.50 \leq z \leq 0)$
$\quad - P(-0.50 \leq z \leq 0)$
$= .4938 - .1915 = .3023$



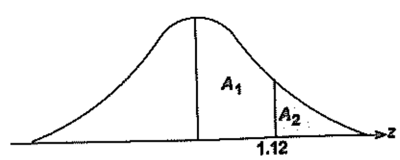
c. $P(13 \leq x \leq 16) = P\left(\frac{13-11}{2} \leq z \leq \frac{16-11}{2}\right)$
$= P(1.00 \leq z \leq 2.50)$
$= P(0 \leq z \leq 2.50)$
$\quad - P(0 \leq z \leq 1.00)$
$= .4938 - .3413 = .1525$



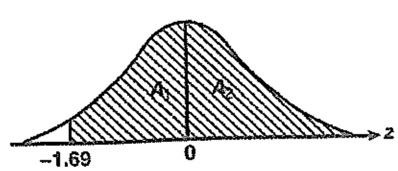
d. $P(7.8 \leq x \leq 12.6)$
$= P\left(\frac{7.8-11}{2} \leq z \leq \frac{12.6-11}{2}\right)$
$= P(-1.60 \leq z \leq 0.80)$
$= A_1 + A_2$
$= .4452 + .2881 = .7333$



e. $P(x \geq 13.24) = P\left(z \geq \frac{13.24-11}{2}\right)$
$= P(z \geq 1.12)$
$= A_2 = .5 - A_1$
$= .5000 - .3686 = .1314$



f. $P(x \geq 7.62) = P\left(z \geq \frac{7.62-11}{2}\right)$
$= P(z \geq -1.69)$
$= A_1 + A_2$
$= .4545 + .5000 = .9545$

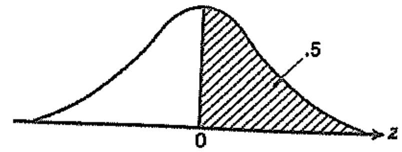


5.32 a. $P(x \geq x_0) = .5 \Rightarrow P\left(z \geq \frac{x_0 - 30}{8}\right)$

$= P(x \geq z_0) = .5$

$\Rightarrow z_0 = 0 = \frac{x_0 - 30}{8}$

$\Rightarrow x_0 = 8(0) + 30 = 30$

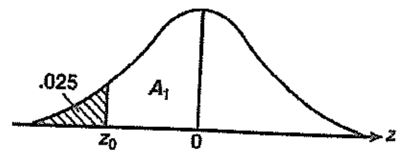


b. $P(x < x_0) = .025 \Rightarrow P\left(z < \frac{x_0 - 30}{8}\right)$

$= P(z < z_0) = .025$

$A_1 = .5 - .025 = .4750$

Looking up the area .4750 in Table IV gives $z_0 = 1.96$. Since z_0 is to the left of 0, $z_0 = -1.96$.



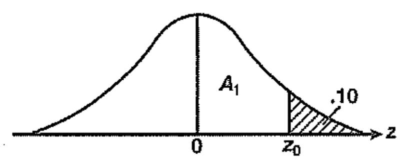
$z_0 = -1.96 = \frac{x_0 - 30}{8} \Rightarrow x_0 = 8(-1.96) + 30 = 14.32$

c. $P(x > x_0) = .10 \Rightarrow P\left(z > \frac{x_0 - 30}{8}\right)$

$= P(z > z_0) = .10$

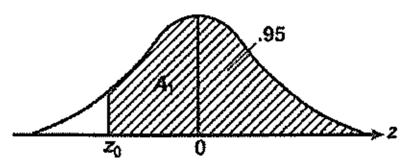
$A_1 = .5 - .10 = .4000$

Looking up the area .4000 in Table IV gives $z_0 = 1.28$.



$z_0 = 1.28 = \frac{x_0 - 30}{8} \Rightarrow x_0 = 8(1.28) + 30 = 40.24$

d. $P(x > x_0) = .95 \Rightarrow P\left(z > \frac{x_0 - 30}{8}\right)$



$= P(z > z_0) = .95$

$A_1 = .95 - .50 = .4500$

Looking up the area .4500 in Table IV gives $z_0 = 1.645$. Since z_0 is to the left of 0, $z_0 = -1.645$.

$z_0 = -1.645 = \frac{x_0 - 30}{8} \Rightarrow x_0 = 8(-1.645) + 30 = 16.84$

5.36 a. Let x = score on Dental Anxiety Scale. Then $z = \frac{x - \mu}{\sigma} = \frac{16 - 11}{3.5} = 1.43$

b. Using Table IV, Appendix A,

$P(10 < x < 15) = P\left(\frac{10-11}{3.5} < z < \frac{15-11}{3.5}\right) = P(-.29 < z < 1.14)$

$= P(-.29 < z < 0) + P(0 < z < 1.14) = .1141 + .3729 = .4870$

c. Using Table IV, Appendix A,

$P(x > 17) = P\left(z > \frac{17-11}{3.5}\right) = P(z > 1.71) = .5 - P(0 < z < 1.71) = .5 - .4564 = .0436$

5.38 a. Let x = change in SAT-MATH score. Using Table IV, Appendix A,

$$P(x \geq 50) = P\left(z \geq \frac{50-19}{65}\right) = P(z \geq .48) = .5 - .1844 = .3156.$$

b. Let x = change in SAT-VERBAL score. Using Table IV, Appendix A,

$$P(x \geq 50) = P\left(z \geq \frac{50-7}{49}\right) = P(z \geq .88) = .5 - .3106 = .1894.$$

5.42 a. Using Table IV, Appendix A, with $\mu = 24.1$ and $\sigma = 6.30$,

$$P(x \geq 20) = P\left(z \geq \frac{20-24.1}{6.30}\right) = P(z \geq -.65) = P(-.65 \leq z \leq 0) + .5$$
$$= .2422 + .5 = .7422$$

b. $$P(x \leq 10.5) = P\left(z \leq \frac{10.5-24.1}{6.30}\right) = P(z \leq -2.16) = .5 - P(-2.16 \leq z \leq 0)$$
$$= .5 - .4846 = .0154$$

c. No. The probability of having a cardiac patient who participates regularly in sports or exercise with a maximum oxygen uptake of 10.5 or smaller is very small ($p = .0154$). It is very unlikely that this patient participates regularly in sports or exercise.

5.44 a. Using Table IV, Appendix A,

$$P(40 < x < 50) = P\left(\frac{40-37.9}{12.4} < z < \frac{50-37.9}{12.4}\right) = P(.17 < z < .98)$$
$$= .3365 - .0675 = .2690.$$

b. Using Table IV, Appendix A,

$$P(x < 30) = P\left(z < \frac{30-37.9}{12.4}\right) = P(z < -.64) = .5 - .2389 = .2611.$$

c. We know that if $P(z_L < z < z_U) = .95$, then $P(z_L < z < 0) + P(0 < z < z_U) = .95$ and

$P(z_L < z < 0) = P(0 < z < z_U) = .95/2 = .4750.$

Using Table IV, Appendix A, $z_U = 1.96$ and $z_L = -1.96$.

$$P(x_L < x < x_U) = .95 \Rightarrow P\left(\frac{x_L - 37.9}{12.4} < z < \frac{x_U - 37.9}{12.4}\right) = .95$$

$$\Rightarrow \frac{x_L - 37.9}{12.4} = -1.96 \quad \text{and} \quad \frac{x_U - 37.9}{12.4} = 1.96$$

$$\Rightarrow x_L - 37.9 = -24.3 \quad \text{and} \quad x_U - 37.9 = 24.3 \quad \Rightarrow x_L = 13.6 \quad \text{and } x_U = 62.2$$

d. $P(z > z_o) = .10 \Rightarrow P(0 < z < z_o) = .4000$. Using Table IV, Appendix A, $z_o = 1.28$.

$$P(x > x_o) = .10 \Rightarrow \frac{x_0 - 37.9}{12.4} = 1.28 \Rightarrow x_o - 37.9 = 15.9 \Rightarrow x_o = 53.8.$$

5.74 a. Using Table II, $P(x \le 11) = .345$

$\mu = np = 25(.5) = 12.5$, $\sigma = \sqrt{npq} = \sqrt{25(.5)(.5)} = 2.5$

Using the normal approximation,

$$P(x \le 11) \approx P\left(z \le \frac{(11+.5)-12.5}{2.5}\right) = P(z \le -.40) = .5 - .1554 = .3446$$

(from Table IV, Appendix A)

b. Using Table II, $P(x \ge 16) = 1 - P(x \le 15) = 1 - .885 = .115$

Using the normal approximation,

$$P(x \ge 16) \approx P\left(z \ge \frac{(16-.5)-12.5}{2.5}\right) = P(z \ge 1.2) = .5 - .3849 = .1151$$

(from Table IV, Appendix A)

c. Using Table II, $P(8 \le x \le 16) = P(x \le 16) - P(x \le 7) = .946 - .022 = .924$

Using the normal approximation,

$$P(8 \le x \le 16) \approx P\left(\frac{(8-.5)-12.5}{2.5} \le z \le \frac{(16+.5)-12.5}{2.5}\right)$$

$$= P(-2.0 \le z \le 1.6) = .4772 + .4452 = .9224$$

(from Table IV, Appendix A)

5.76 $\mu = np = 1000(.5) = 500$, $\sigma = \sqrt{npq} = \sqrt{1000(.5)(.5)} = 15.811$

a. Using the normal approximation,

$$P(x > 500) \approx P\left(z > \frac{(500+.5)-500}{15.811}\right) = P(z > .03) = .5 - .0120 = .4880$$

(from Table IV, Appendix A)

b. $$P(490 \le x < 500) \approx P\left(\frac{(490-.5)-500}{15.811} \le z < \frac{(500-.5)-500}{15.811}\right)$$

$$= P(-.66 \le z < -.03) = .2454 - .0120 = .2334$$

(from Table IV, Appendix A)

c. $$P(x > 550) \approx P\left(z > \frac{(500+.5)-500}{15.811}\right) = P(z > 3.19) \approx .5 - .5 = 0$$

(from Table IV, Appendix A)

5.84 a. Let x = number of abused women in a sample of 150. The random variable x is a binomial random variable with $n = 150$ and $p = 1/3$. Thus, for the normal approximation,

$$\mu = np = 150(1/3) = 50 \text{ and } \sigma = \sqrt{npq} = \sqrt{150(1/3)(2/3)} = 5.7735$$

$$\mu \pm 3\sigma \Rightarrow 50 \pm 3(5.7735) \Rightarrow 50 \pm 17.3205 \Rightarrow (32.6795, 67.3205)$$

Since this interval lies in the range from 0 to $n = 150$, the normal approximation is appropriate.

$$P(x > 75) \approx P\left(z > \frac{(75+.5)-50}{5.7735}\right) = P(z > 4.42) \approx .5 - .5 = 0$$

(Using Table IV, Appendix A.)

b. $$P(x < 50) \approx P\left(z < \frac{(50-.5)-50}{5.7735}\right) = P(z < -.09) \approx .5 - .0359 = .4641$$

c. $$P(x < 30) \approx P\left(z < \frac{(30-.5)-50}{5.7735}\right) = P(z < -3.55) \approx .5 - .5 = 0$$

Since the probability of seeing fewer than 30 abused women in a sample of 150 is so small ($p \approx 0$), it would be very unlikely to see this event.

5.88 Let x = number of patients who wait more than 20 minutes. Then x is a binomial random variable with $n = 150$ and $p = .5$.

a. $\mu = np = 150(.5) = 75$, $\sigma = \sqrt{npq} = \sqrt{150(.5)(.5)} = 6.124$

$$P(x > 75) \approx P\left(z > \frac{(75+.5)-75}{6.124}\right) = P(z > .08) = .5 - .0319 = .4681$$

(from Table IV, Appendix A)

b. $$P(x > 85) \approx P\left(z > \frac{(85+.5)-75}{6.124}\right) = P(z > 1.71) = .5 - .4564 = .0436$$

(from Table IV, Appendix A)

c. $$P(60 < x < 90) \approx P\left(\frac{(60+.5)-75}{6.124} < z < \frac{(90-.5)-75}{6.124}\right)$$

$$= P(-2.37 < z < 2.37) = .4911 + .4911 = .9822$$

(from Table IV, Appendix A)

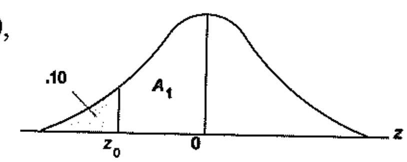
5.116 a. Using Table IV, Appendix A, with $\mu = 8.72$ and $\sigma = 1.10$,

$$P(x < 6) = P\left(z < \frac{6 - 8.72}{1.10}\right) = P(z < -2.47) = .5 - .4932 = .0068$$

Thus, approximately .68% of the games would result in fewer than 6 hits.

b. The probability of observing fewer than 6 hits in a game is $p = .0068$. The probability of observing 0 hits would be even smaller. Thus, it would be extremely unusual to observe a no hitter.

5.130 a. Using Table IV, Appendix A, with $\mu = 450$ and $\sigma = 40$,



$$P(x < x_0) = .10 \Rightarrow P\left(z < \frac{x_0 - 450}{40}\right)$$

$$= P(z < z_0) = .10$$

$A_1 = .5 - .10 = .4000$

Looking up the area .4000 in Table IV gives $z_0 = 1.28$. Since z_0 is to the left of 0, $z_0 = -1.28$.

$$z_0 = -1.28 = \frac{x_0 - 450}{40} \Rightarrow x_0 = 40(-1.28) + 450 = 398.8 \text{ seconds.}$$