

STAT 213 L05

①

Solutions to Assign #6

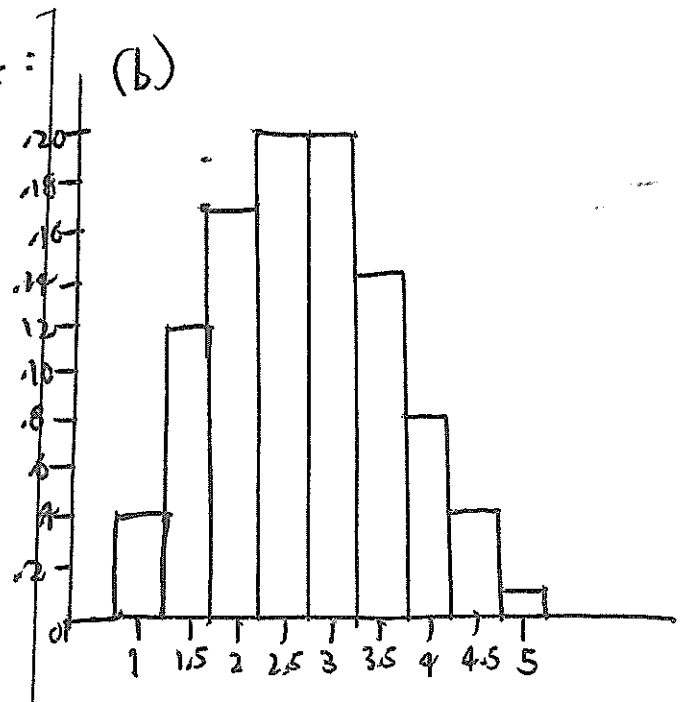
6.5 (a)

sample	\bar{X}	prob
1,1	1	.04
1,2	1.5	.06
1,3	2	.04
1,4	2.5	.04
1,5	3	.02
1,5		
2,1	1.5	.06
2,2	2	.09
2,3	2.5	.06
2,4	3	.06
2,5	3.5	.03
3,1	2	.04
3,2	2.5	.06
3,3	3	.04

sample	\bar{X}	prob
3,4	3.5	.04
3,5	4	.02
4,1	2.5	.04
4,2	3	.06
4,3	3.5	.04
4,4	4	.04
4,5	4.5	.02
5,1	3	.02
5,2	3.5	.03
5,3	4	.02
5,4	4.5	.02
5,5	5	.01

So sampling distv. of \bar{X} is:

\bar{X}	$P[\bar{X}=\bar{x}]$
1	.04
1.5	.12
2	.17
2.5	.20
3	.20
3.5	.14
4	.08
4.5	.04
5	.01
$\Sigma =$	1.00 ✓



6.6 $E(x) = \mu = \sum xp(x) = 1(.2) + 2(.3) + 3(.2) + 4(.2) + 5(.1)$
 $= .2 + .6 + .6 + .8 + .5 = 2.7$

$E(\bar{x}) = \sum \bar{x}p(\bar{x}) = 1.0(.04) + 1.5(.12) + 2.0(.17) + 2.5(.20) + 3.0(.20) + 3.5(.14) + 4.0(.08)$
 $+ 4.5(.04) + 5.0(.01)$
 $= .04 + .18 + .34 + .50 + .60 + .49 + .32 + .18 + .05 = 2.7$

6.10 A point estimator of a population parameter is a rule or formula that tells us how to use the sample data to calculate a single number that can be used as an *estimate* of the population parameter.

6.12 The MVUE is the minimum variance unbiased estimator. The MUVE for a parameter is an unbiased estimator of the parameter that has the minimum variance of all unbiased estimators.

6.14 a. $\mu = \sum xp(x) = 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right) = \frac{5}{3} = 1.667$

$\sigma^2 = \sum (x - \mu)^2 p(x) = \left(0 - \frac{5}{3}\right)^2 \left(\frac{1}{3}\right) + \left(1 - \frac{5}{3}\right)^2 \left(\frac{1}{3}\right) + \left(4 - \frac{5}{3}\right)^2 \left(\frac{1}{3}\right)$
 $= \frac{78}{27} = 2.889$

b.

Sample	\bar{x}	Probability
0, 0	0	1/9
0, 1	0.5	1/9
0, 4	2	1/9
1, 0	0.5	1/9
1, 1	1	1/9
1, 4	2.5	1/9
4, 0	2	1/9
4, 1	2.5	1/9
4, 4	4	1/9

(continued)

6.14 (continued)

\bar{x}	Probability
0	1/9
0.5	2/9
1	1/9
2	2/9
2.5	2/9
4	1/9

c.
$$E(\bar{x}) = \sum \bar{x}p(\bar{x}) = 0\left(\frac{1}{9}\right) + 0.5\left(\frac{2}{9}\right) + 1\left(\frac{1}{9}\right) + 2\left(\frac{2}{9}\right) + 2.5\left(\frac{2}{9}\right) + 4\left(\frac{1}{9}\right)$$

$$= \frac{15}{9} = \frac{5}{3} = 1.667$$

Since $E(\bar{x}) = \mu$, \bar{x} is an unbiased estimator for μ .

d.

Sample	s^2	Probability
0, 0	0	1/9
0, 1	0.5	1/9
0, 4	8	1/9
1, 0	0.5	1/9
1, 1	0	1/9
1, 4	4.5	1/9
4, 0	8	1/9
4, 1	4.5	1/9
4, 4	0	1/9

s^2	Probability
0	3/9
0.5	2/9
4.5	2/9
8	2/9

e.
$$E(s^2) = \sum s^2 p(s^2) = 0\left(\frac{3}{9}\right) + 0\left(\frac{2}{9}\right) + 4.5\left(\frac{2}{9}\right) + 8\left(\frac{2}{9}\right) = \frac{26}{9} = 2.889$$

Since $E(s^2) = \sigma^2$, s^2 is an unbiased estimator for σ^2 .

6.16 a.
$$\mu = \sum xp(x) = 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{3}\right) + 2\left(\frac{1}{3}\right) = 1$$

(continued)

6/16 (continued)

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b.

Sample	Probability	Sample	Probability		
0, 0, 0	0	1/27	1, 1, 2	4/3	1/27
0, 0, 1	1/3	1/27	1, 2, 0	1	1/27
0, 0, 2	2/3	1/27	1, 2, 1	4/3	1/27
0, 1, 0	1/3	1/27	1, 2, 2	5/3	1/27
0, 1, 1	2/3	1/27	2, 0, 0	2/3	1/27
0, 1, 2	1	1/27	2, 0, 1	1	1/27
0, 2, 0	2/3	1/27	2, 0, 2	4/3	1/27
0, 2, 1	1	1/27	2, 1, 0	1	1/27
0, 2, 2	4/3	1/27	2, 1, 1	4/3	1/27
1, 0, 0	1/3	1/27	2, 1, 2	5/3	1/27
1, 0, 1	2/3	1/27	2, 2, 0	4/3	1/27
1, 0, 2	1	1/27	2, 2, 1	5/3	1/27
1, 1, 0	2/3	1/27	2, 2, 2	2	1/27
1, 1, 1	1	1/27			

From the above table, the sampling distribution of the sample mean would be:

\bar{x}	Probability
0	1/27
1/3	3/27
2/3	6/27
1	7/27
4/3	6/27
5/3	3/27
2	1/27

c.

Sample	m	Probability	Sample	m	Probability
0, 0, 0	0	1/27	1, 1, 2	1	1/27
0, 0, 1	0	1/27	1, 2, 0	1	1/27
0, 0, 2	0	1/27	1, 2, 1	1	1/27
0, 1, 0	0	1/27	1, 2, 2	2	1/27
0, 1, 1	1	1/27	2, 0, 0	0	1/27
0, 1, 2	1	1/27	2, 0, 1	1	1/27
0, 2, 0	0	1/27	2, 0, 2	2	1/27
0, 2, 1	1	1/27	2, 1, 0	1	1/27
0, 2, 2	2	1/27	2, 1, 1	1	1/27
1, 0, 0	0	1/27	2, 1, 2	2	1/27
1, 0, 1	1	1/27	2, 2, 0	2	1/27
1, 0, 2	1	1/27	2, 2, 1	2	1/27
1, 1, 0	1	1/27	2, 2, 2	2	1/27
1, 1, 1	1	1/27			

(continued)

816 (continued)

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From the above table, the sampling distribution of the sample median would be:

m	Probability
0	$7/27$
1	$13/27$
2	$7/27$

d.
$$E(\bar{x}) = \sum \bar{x}p(\bar{x}) = 0\left(\frac{1}{27}\right) + \frac{1}{3}\left(\frac{3}{27}\right) + \frac{2}{3}\left(\frac{6}{27}\right) + 1\left(\frac{7}{27}\right) + \frac{4}{3}\left(\frac{6}{27}\right) + \frac{5}{3}\left(\frac{3}{27}\right) + 2\left(\frac{1}{27}\right)$$
$$= 1$$

Since $E(\bar{x}) = \mu$, \bar{x} is an unbiased estimator for μ .

$$E(m) = \sum mp(m) = 0\left(\frac{7}{27}\right) + 1\left(\frac{13}{27}\right) + 2\left(\frac{7}{27}\right) = 1$$

Since $E(m) = \mu$, m is an unbiased estimator for μ .

e.
$$\sigma_{\bar{x}}^2 = \sum (\bar{x} - \mu)^2 p(\bar{x})$$
$$= (0-1)^2\left(\frac{1}{27}\right) + \left(\frac{1}{3}-1\right)^2\left(\frac{3}{27}\right) + \left(\frac{2}{3}-1\right)^2\left(\frac{6}{27}\right) + (1-1)^2\left(\frac{7}{27}\right)$$
$$+ \left(\frac{4}{3}-1\right)^2\left(\frac{6}{27}\right) + \left(\frac{5}{3}-1\right)^2\left(\frac{3}{27}\right) + (2-1)^2\left(\frac{1}{27}\right) = \frac{2}{9} = .2222$$

$$\sigma_m^2 = \sum (m-1)^2 p(m) = (0-1)^2\left(\frac{7}{27}\right) + (1-1)^2\left(\frac{13}{27}\right) + (2-1)^2\left(\frac{7}{27}\right) = \frac{14}{27}$$
$$= .5185$$

f. Since both the sample mean and median are unbiased and the variance is smaller for the sample mean, it would be the preferred estimator of μ .

6.30

a. $\mu_{\bar{x}} = \mu = 20$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 16/\sqrt{64} = 2$

b. By the Central Limit Theorem, the distribution of \bar{x} is approximately normal. In order for the Central Limit Theorem to apply, n must be sufficiently large. For this problem, $n = 64$ is sufficiently large.

c.
$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{16 - 20}{2} = -2.00$$

d.
$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{23 - 20}{2} = 1.50$$

e.
$$P(\bar{x} < 16) = P\left(z < \frac{16 - 20}{2}\right) = P(z < -2) = .5 - .4772 = .0228$$

f.
$$P(\bar{x} > 23) = P\left(z > \frac{23 - 20}{2}\right) = P(z > 1.50) = .5 - .4332 = .0668$$

g.
$$P(16 < \bar{x} < 22) = P\left(\frac{16 - 20}{2} < z < \frac{22 - 20}{2}\right) = P(-2 < z < 1)$$
$$= .4772 + .3413 = .8185$$

- 6.34
- a. $\mu_{\bar{x}}$ is the mean of the sampling distribution of \bar{x} . $\mu_{\bar{x}} = \mu = 106$.
 - b. $\sigma_{\bar{x}}$ is the standard deviation of the sampling distribution of \bar{x} . $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16.4}{\sqrt{36}} = 2.73$
 - c. By the Central Limit Theorem, the sampling distribution of \bar{x} is approximately normal.
 - d. $z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{100 - 106}{2.73} = -2.20$
 - e. $P(\bar{x} < 100) = P(z < -2.20) = .5 - .4864 = .0136$ (Using Table IV, Appendix A.)

- 6.36
- a. Let \bar{x} = sample mean FNE score. By the Central Limit Theorem, the sampling distribution of \bar{x} is approximately normal with

$$\mu_{\bar{x}} = \mu = 18 \text{ and } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{45}} = .7453.$$

$$P(\bar{x} > 17.5) = P\left(z > \frac{17.5 - 18}{.7453}\right) = P(z > -.67) = .5 + .2486 = .7486$$

(Using Table IV, Appendix A)
 - b. $P(18 < \bar{x} < 18.5) = P\left(\frac{18 - 18}{.7453} < z < \frac{18.5 - 18}{.7453}\right) = P(0 < z < .67) = .2486$

(Using Table IV, Appendix A)
 - c. $P(\bar{x} < 18.5) = P\left(z < \frac{18.5 - 18}{.7453}\right) = P(z < .67) \Rightarrow .5 + .2486 = .7486$

(Using Table IV, Appendix A)

- 7.10
- a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\bar{x} \pm z_{.025} \frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.96 \frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .56 \Rightarrow (25.34, 26.46)$$
 - b. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The confidence interval is:

$$\bar{x} \pm z_{.05} \frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 1.645 \frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .47 \Rightarrow (25.43, 26.37)$$
 - c. For confidence coefficient .99, $\alpha = .01$ and $\alpha/2 = .01/2 = .005$. From Table IV, Appendix A, $z_{.005} = 2.58$. The confidence interval is:

$$\bar{x} \pm z_{.005} \frac{s}{\sqrt{n}} \Rightarrow 25.9 \pm 2.58 \frac{2.7}{\sqrt{90}} \Rightarrow 25.9 \pm .73 \Rightarrow (25.17, 26.63)$$

- 7.12 a. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\bar{x} \pm z_{.025} \frac{s}{\sqrt{n}} \Rightarrow 33.9 \pm 1.96 \frac{3.3}{\sqrt{100}} \Rightarrow 33.9 \pm .647 \Rightarrow (33.253, 34.547)$$

b. $\bar{x} \pm z_{.025} \frac{s}{\sqrt{n}} \Rightarrow 33.9 \pm 1.96 \frac{3.3}{\sqrt{400}} \Rightarrow 33.9 \pm .323 \Rightarrow (33.577, 34.223)$

- c. For part a, the width of the interval is $2(.647) = 1.294$. For part b, the width of the interval is $2(.323) = .646$. When the sample size is quadrupled, the width of the confidence interval is halved.

- 7.14 a. The point estimate for the mean personal network size of all older adults is $\bar{x} = 14.6$.

- b. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The 95% confidence interval is:

$$\bar{x} \pm z_{.025} \frac{\sigma}{\sqrt{n}} \Rightarrow \bar{x} \pm 1.96 \frac{9.8}{\sqrt{2,819}} \Rightarrow 14.6 \pm .36 \Rightarrow (14.24, 14.96)$$

- c. We are 95% confident that the mean personal network size of all older adults is between 14.24 and 14.96.
- d. We must assume that we have a random sample from the target population and that the sample size is sufficiently large.

- 7.18 a. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The confidence interval is:

$$\bar{x} \pm z_{.05} \frac{s}{\sqrt{n}} \Rightarrow 7.62 \pm 1.645 \frac{8.91}{\sqrt{65}} \Rightarrow 7.62 \pm 1.82 \Rightarrow (5.80, 9.44)$$

- b. We are 90% confident that the mean sentence complexity score of all low-income children is between 5.80 and 9.44.
- c. Yes. We are 90% confident that the mean sentence complexity score of all low-income children is between 5.80 and 9.44. Since the mean score for middle-income children, 15.55, is outside this interval, there is evidence that the true mean for low-income children is different from 15.55.

7.32

For this sample,

$$\bar{x} = \frac{\sum x}{n} = \frac{1567}{16} = 97.9375$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = \frac{155,867 - \frac{1567^2}{16}}{16-1} = 159.9292$$

$$s = \sqrt{s^2} = 12.6463$$

- a. For confidence coefficient, .80, $\alpha = 1 - .80 = .20$ and $\alpha/2 = .20/2 = .10$. From Table VI, Appendix A, with $df = n - 1 = 16 - 1 = 15$, $t_{.10} = 1.341$. The 80% confidence interval for μ is:

$$\bar{x} \pm t_{.10} \frac{s}{\sqrt{n}} \Rightarrow 97.94 \pm 1.341 \frac{12.6463}{\sqrt{16}} \Rightarrow 97.94 \pm 4.240 \Rightarrow (93.700, 102.180)$$

- b. For confidence coefficient, .95, $\alpha = 1 - .95 = .05$ and $\alpha/2 = .05/2 = .025$. From Table VI, Appendix A, with $df = n - 1 = 16 - 1 = 15$, $t_{.025} = 2.131$. The 95% confidence interval for μ is:

$$\bar{x} \pm t_{.025} \frac{s}{\sqrt{n}} \Rightarrow 97.94 \pm 2.131 \frac{12.6463}{\sqrt{16}} \Rightarrow 97.94 \pm 6.737 \Rightarrow (91.203, 104.677)$$

The 95% confidence interval for μ is wider than the 80% confidence interval for μ found in part a.

- c. For part a:

We are 80% confident that the true population mean lies in the interval 93.700 to 102.180.

For part b:

We are 95% confident that the true population mean lies in the interval 91.203 to 104.677.

The 95% confidence interval is wider than the 80% confidence interval because the more confident you want to be that μ lies in an interval, the wider the range of possible values.

7.34

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table VI, Appendix A, with $df = n - 1 = 25 - 1 = 24$, $t_{.05} = 1.711$. The 90% confidence interval is:

$$\bar{x} \pm t_{.05,24} \frac{s}{\sqrt{n}} \Rightarrow 75.4 \pm 1.711 \frac{10.9}{\sqrt{25}} \Rightarrow 75.4 \pm 3.73 \Rightarrow (71.67, 79.13)$$

We are 90% confident that the true mean breaking strength of white wood is between 71.67 and 79.13 MPa's.

7.36

- a. The point estimate for the average annual rainfall amount at ant sites in the Dry Steppe region of Central Asia is $\bar{x} = 183.4$ milliliters.
- b. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table VI, Appendix A, with $df = n - 1 = 5 - 1 = 4$, $t_{.05} = 2.132$.
- c. The 90% confidence interval is:

$$\bar{x} \pm t_{.05} \frac{s}{\sqrt{n}} \Rightarrow 183.4 \pm 2.132 \frac{20.6470}{\sqrt{5}} \Rightarrow 183.4 \pm 19.686 \Rightarrow (163.714, 203.086)$$

- d. We are 90% confident that the average annual rainfall amount at ant sites in the Dry Steppe region of Central Asia is between 163.714 and 203.086 milliliters.
- e. Using MINITAB, the 90% confidence interval is:

One-Sample T: DS Rain

Variable	N	Mean	StDev	SE Mean	90% CI
DS Rain	5	183.400	20.647	9.234	(163.715, 203.085)

The 90% confidence interval is (163.715, 203.085). This is very similar to the confidence interval calculated in part c.

- f. The point estimate for the average annual rainfall amount at ant sites in the Gobi Desert region of Central Asia is $\bar{x} = 110.0$ milliliters.

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table VI, Appendix A, with $df = n - 1 = 6 - 1 = 5$, $t_{.05} = 2.015$.

The 90% confidence interval is:

$$\bar{x} \pm t_{.05} \frac{s}{\sqrt{n}} \Rightarrow 110.0 \pm 2.015 \frac{15.975}{\sqrt{6}} \Rightarrow 110.0 \pm 13.141 \Rightarrow (96.859, 123.141)$$

We are 90% confident that the average annual rainfall amount at ant sites in the Gobi Desert region of Central Asia is between 96.859 and 123.141 milliliters.

Using MINITAB, the 90% confidence interval is:

One-Sample T: GD Rain

Variable	N	Mean	StDev	SE Mean	90% CI
GD Rain	6	110.000	15.975	6.522	(96.858, 123.142)

The 90% confidence interval is (96.858, 123.142). This is very similar to the confidence interval calculated above.

- 7.48 a. The sample size is large enough if both $n\hat{p} \geq 15$ and $n\hat{q} \geq 15$.

$n\hat{p} = 144(.76) = 109.44$ and $n\hat{q} = 144(.24) = 34.56$. Since both of these numbers are greater than or equal to 15, the sample size is sufficiently large to conclude the normal approximation is reasonable.

- b. For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The 90% confidence interval is:

$$\hat{p} \pm z_{.05} \sqrt{\frac{pq}{n}} \approx \hat{p} \pm 1.645 \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .76 \pm 1.645 \sqrt{\frac{.76(.24)}{144}} \Rightarrow .76 \pm .059 \\ = (.701, .819)$$

- c. We must assume the sample was randomly selected from the population of interest. We must also assume our sample size is sufficiently large to ensure the sampling distribution is approximately normal. From the results of part a, this appears to be a reasonable assumption.

- 7.50 a. Of the 50 observations, 15 like the product $\Rightarrow \hat{p} = \frac{15}{50} = .30$.

The sample size is large enough if both $n\hat{p} \geq 15$ and $n\hat{q} \geq 15$.

$n\hat{p} = 50(.3) = 15$ and $n\hat{q} = 50(.7) = 35$. Since both of these numbers are greater than or equal to 15, the sample size is sufficiently large to conclude the normal approximation is reasonable.

For the confidence coefficient .80, $\alpha = .20$ and $\alpha/2 = .10$. From Table IV, Appendix A, $z_{.10} = 1.28$. The confidence interval is:

$$\hat{p} \pm z_{.10} \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .3 \pm 1.28 \sqrt{\frac{.3(.7)}{50}} \Rightarrow .3 \pm .083 \Rightarrow (.217, .383)$$

- b. We are 80% confident the proportion of all consumers who like the new snack food is between .217 and .383.

- 7.54 a. The point estimate of p is $\hat{p} = x/n = 39/150 = .26$.

- b. The sample size is large enough if both $n\hat{p} \geq 15$ and .

$n\hat{p} = 150(.26) = 39$ and $n\hat{q} = 150(.74) = 111$. Since both of the numbers are greater than or equal to 15, the sample size is sufficiently large to conclude the normal approximation is reasonable.

For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .05/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\hat{p} \pm z_{.025} \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .26 \pm 1.96 \sqrt{\frac{.26(.74)}{150}} \Rightarrow .26 \pm .070 \Rightarrow (.190, .330)$$

- c. We are 95% confident that the true proportion of college students who experience "residual anxiety" from a scary TV show or movie is between .190 and .330.

7.58 a. First, we compute \hat{p} : $\hat{p} = \frac{x}{n} = \frac{15}{40} = .375$

The sample size is large enough if both $n\hat{p} \geq 15$ and $n\hat{q} \geq 15$.

$n\hat{p} = 40(.375) = 15$ and $n\hat{q} = 40(.625) = 25$. Since both of the numbers are greater than or equal to 15, the sample size is sufficiently large to conclude the normal approximation is reasonable.

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The 90% confidence interval is:

$$\begin{aligned} \hat{p} \pm z_{.05} \sqrt{\frac{pq}{n}} &\Rightarrow \hat{p} \pm 1.645 \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .375 \pm 1.645 \sqrt{\frac{.375(.625)}{40}} \\ &\Rightarrow .375 \pm .126 \Rightarrow (.249, .501) \end{aligned}$$

We are 90% confident that the true dropout rate for exercisers who vary their routine in workouts is between .249 and .501.

b. First, we compute \hat{p} : $\hat{p} = \frac{x}{n} = \frac{23}{40} = .575$

The sample size is large enough if both $n\hat{p} \geq 15$ and $n\hat{q} \geq 15$.

$n\hat{p} = 40(.575) = 23$ and $n\hat{q} = 40(.425) = 17$. Since both of the numbers are greater than or equal to 15, the sample size is sufficiently large to conclude the normal approximation is reasonable.

For confidence coefficient .90, $\alpha = .10$ and $\alpha/2 = .10/2 = .05$. From Table IV, Appendix A, $z_{.05} = 1.645$. The 90% confidence interval is:

$$\begin{aligned} \hat{p} \pm z_{.05} \sqrt{\frac{pq}{n}} &\Rightarrow \hat{p} \pm 1.645 \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .575 \pm 1.645 \sqrt{\frac{.575(.425)}{40}} \\ &\Rightarrow .575 \pm .129 \Rightarrow (.446, .704) \end{aligned}$$

We are 90% confident that the true dropout rate for exercisers who have no set schedule for their workouts is between .446 and .704.