STAT 213 L05

Solutions to Assignment No. 7

8.22 a. $z = \frac{\bar{x}-\mu}{\sigma_{\bar{x}}} = \frac{215-200}{80/\sqrt{100}} = 1.88$

The decision rule is: Reject H_0 if $z > 1.88$.

b. $\alpha = P(z > 1.88) = .5 - .4699 = .0301$ (using Table IV, Appendix A)

8.24 a. H_0: $\mu = .36$
H_a: $\mu < .36$

The test statistic is $z = \frac{\bar{x}-\mu_0}{\sigma_{\bar{x}}} = \frac{.323-.36}{\sqrt{.034}/\sqrt{64}} = -1.61$

The rejection region requires $\alpha = .10$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.10} = 1.28$. The rejection region is $z < -1.28$.

Since the observed value of the test statistic falls in the rejection region ($z = -1.61 < -1.28$), H_0 is rejected. There is sufficient evidence to indicate the mean is less than .36 at $\alpha = .10$.

b. H_0: $\mu = .36$
H_a: $\mu \neq .36$

The test statistic is $z = -1.61$ (see part **a**).

The rejection region requires $\alpha/2 = .10/2 = .05$ in the each tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$ or $z > 1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.61 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate the mean is different from .36 at $\alpha = .10$.

8.26 a. The rejection region requires $\alpha = .01$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.01} = 2.33$. The rejection region is $z < -2.33$.

b. The test statistic is $z = \frac{\bar{x}-\mu_0}{\sigma_{\bar{x}}} = \frac{19.3-20}{\frac{11.9}{\sqrt{46}}} = -.40$

c. Since the observed value of the test statistic does not fall in the rejection region ($z = -.40 \not< -2.33$), H_0 is not rejected. There is insufficient evidence to indicate the mean number of latex gloves used per week by hospital employees diagnosed with a latex allergy from exposure to the powder on latex gloves is less than 20 at $\alpha = .01$.

8.28 To determine if the area sampled is grassland, we test:

H_0: $\mu = 220$
H_a: $\mu \neq 220$

The test statistic is $z = \frac{\bar{x} - \mu_o}{\sigma_{\bar{x}}} = \frac{225 - 220}{\frac{20}{\sqrt{100}}} = 2.50$

The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.005} = 2.575$. The rejection region is $z > 2.575$ or $z < -2.575$.
Since the observed value of the test statistic does not fall in the rejection region ($z = 2.50 \not> 2.575$). H_0 is not rejected. There is insufficient evidence to indicate the true mean lacunarity measurements is different from 220 at $\alpha = .01$. There is insufficient evidence to indicate that the sampled area is not grassland.

8.30 To determine if the mean point-spread error is different from 0, we test:

H_0: $\mu = 0$
H_a: $\mu \neq 0$

The test statistic is $z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{-1.6 - 0}{13.3/\sqrt{240}} = -1.86$

The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.005} = 2.575$. The rejection region is $z > 2.575$ or $z < -2.575$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -1.86 \not< -2.575$), H_0 is not rejected. There is insufficient evidence to indicate that the true mean point-spread error is different from 0 at $\alpha = .01$.

8.40 The smallest value of α for which the null hypothesis would be rejected is just greater than .06.

8.42 p-value $= P(z \geq 2.17) + P(z \leq -2.17) = (.5 - .4850)2 = .0300$ (using Table IV, Appendix A)

8.44 First, find the value of the test statistic:

$$z = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{10.7 - 10}{3.1/\sqrt{50}} = 1.60$$

p-value $= P(z \leq -1.60 \text{ or } z \geq 1.60) = 2P(z \geq 1.60) = 2(.5 - .4452) = 2(.0548) = .1096$

There is no evidence to reject H_0 for $\alpha \leq .10$.

8.46 a. From Exercise 8.25, $z = -85.52$. The p-value is $p = P(z \leq -85.52) + P(z \geq 85.52) = (.5 - .5) + (.5 - .5) = 0$ (using Table IV, Appendix A).

b. The p-value is $p = 0$. Since this is less than $\alpha = .10$, H_0 is rejected. There is sufficient evidence to indicate the mean response for the population of all New York City public school children different from 3 at $\alpha = .10$.

8.50 a. If chickens are more apt to peck at white string, then they are less apt to peck at blue string. Let μ = mean number of pecks at a blue string. To determine if chickens are more apt to peck at white string than blue string (or less apt to peck at blue string), we test:

H_0: $\mu = 7.5$
H_a: $\mu < 7.5$

The test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \dfrac{1.13 - 7.5}{\frac{2.21}{\sqrt{72}}} = -24.46$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic falls in the rejection region ($z = -24.46 < -1.645$), H_0 is rejected. There is sufficient evidence to indicate the chickens are less apt to peck at blue string at $\alpha = .05$.

b. In Exercise 7.19 b, we concluded that the birds were more apt to peck at white string. The mean number of pecks for white string is 7.5. Since 7.5 is not in the 99% confidence interval for the mean number of pecks at blue string, it is not a likely value for the true mean for blue string.

c. The p-value is $P(z \leq -24.46) = .5 - .5 \approx 0$.

8.60 For this sample,

$$\bar{x} = \frac{\sum x}{n} = \frac{11}{6} = 1.8333 \qquad s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{41 - \frac{11^2}{6}}{6-1} = 4.1667$$

$$s = \sqrt{s^2} = 2.0412$$

a. H_0: $\mu = 3$
H_a: $\mu < 3$
The test statistic is $t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{1.8333 - 3}{2.0412/\sqrt{6}} = -1.40$

The rejection region requires α =.05 in the lower tail of the t distribution with df = n – 1 = 6 – 1 = 5. From Table VI, Appendix A, $t_{.05} = 2.015$. The rejection region is $t < -2.015$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.40 \not< -2.015$), H_0 is not rejected. There is insufficient evidence to indicate μ is less than 3 at $\alpha = .05$.

b. H_0: $\mu = 3$
H_a: $\mu \neq 3$
Test statistic: $t = -1.40$ (Refer to part **a**.)

The rejection region requires $\alpha/2$ =.05/2 = .025 in each tail of the t distribution with df = n – 1 = 6 – 1 = 5. From Table VI, Appendix A, $t_{.025} = 2.571$. The rejection region is $t < -2.571$ or $t > 2.571$.

Since the observed value of the test statistic does not fall in the rejection region ($t = -1.40 \not< -2.571$), H_0 is not rejected. There is insufficient evidence to indicate μ differs from 3 at $\alpha = .05$.

c. For part **a**: p-value = $P(t \le -1.40)$

From Table VI, with df = 5, $P(t \le -1.40) > .10$

For part **b**: p-value = $P(t \le -1.40) + P(t \ge 1.40)$

From Table VI, with df = 5, p-value = $2P(t \ge 1.40) > 2(.10) = .20$

8.66 a. The parameter of interest is μ = mean chromatic contrast of crab-spiders on daisies.

b. To determine if the mean chromatic contrast of crab-spiders on daisies is less than 70, we test:

H_0: $\mu = 70$
H_a: $\mu < 70$

c. $$\bar{x} = \frac{\sum x}{n} = \frac{575}{10} = 57.5$$

$$s^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \frac{42,649 - \frac{575^2}{10}}{10-1} = \frac{9,586.5}{9} = 1,065.1667$$

$$s = \sqrt{1,065.1667} = 32.6369$$

The test statistic is $t = \frac{\bar{x} - \mu_o}{s/\sqrt{n}} = \frac{57.5 - 70}{\frac{32.6369}{\sqrt{10}}} = -1.21$

d. The rejection region requires $\alpha = .10$ in the lower tail of the t-distribution with df $= n - 1 = 10 - 1 = 9$. From Table VI, Appendix A, $t_{.10} = 1.383$. The rejection region is $t < -1.383$.

e. Since the observed value of the test statistic does not fall in the rejection region ($t = -1.21 \not< -1.383$), H_0 is not rejected. There is insufficient evidence to indicate that the mean chromatic contrast of crab-spiders on daisies is less than 70 at $\alpha = .10$.

8.76 b. In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 100(.75) = 75$ and $nq_o = 100(1 - .75) = 100(.25) = 25$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

H_0: $p = .75$
H_a: $p < .75$

The test statistic is $z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.69 - .75}{\sqrt{\frac{.75(.25)}{100}}} = -1.39$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($-1.39 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is less than .75 at $\alpha = .05$.

c. p-value $= P(z \le -1.39) = .5 - .4177 = .0823$

8.78 a. H_0: $p = .65$
H_a: $p > .65$

The test statistic is $z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.74 - .65}{\sqrt{\frac{.65(.35)}{100}}} = 1.89$

The rejection region requires $\alpha = .01$ in the upper tail of the z distribution. From Table IV, Appendix A, $z_{.01} = 2.33$. The rejection region is $z > 2.33$.

Since the observed value of the test statistic does not fall in the rejection region ($1.89 \not> 2.33$), H_0 is not rejected. There is insufficient evidence to indicate that the proportion is greater than .65 at $\alpha = .01$.

b. H_0: $p = .65$
H_a: $p > .65$

The test statistic is $z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.74 - .65}{\sqrt{\frac{.65(.35)}{100}}} = 1.89$

The rejection region requires $\alpha = .10$ in the upper tail of the z distribution. From Table IV, Appendix A, $z_{.10} = 1.28$. The rejection region is $z > 1.28$.

Since the observed value of the test statistic falls in the rejection region ($1.89 > 1.28$), H_0 is rejected. There is sufficient evidence to indicate that the proportion is greater than .65 at $\alpha = .10$.

c. H_0: $p = .90$
H_a: $p \neq .90$

The test statistic is $z = \frac{\hat{p} - p_0}{\sigma_{\hat{p}}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.74 - .90}{\sqrt{\frac{.90(.10)}{100}}} = -5.33$

The rejection region requires $\alpha/2 = .05/2 = .025$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.025} = 1.96$. The rejection region is $z < -1.96$ or $z > 1.96$.

Since the observed value of the test statistic falls in the rejection region ($-5.33 < -1.96$), H_0 is rejected. There is sufficient evidence to indicate that the proportion is different from .90 at $\alpha = .05$.

d. For confidence coefficient .95, $\alpha = .05$ and $\alpha/2 = .025$. From Table IV, Appendix A, $z_{.025} = 1.96$. The confidence interval is:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .74 \pm 1.96\sqrt{\frac{(.74)(.26)}{100}} \Rightarrow .74 \pm .09 \Rightarrow (.65, .83)$$

e. For confidence coefficient .99, $\alpha = .01$ and $\alpha/2 = .005$. From Table IV, Appendix A, $z_{.005} = 2.58$. The confidence interval is:

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow .74 \pm 2.58\sqrt{\frac{(.74)(.26)}{100}} \Rightarrow .74 \pm .11 \Rightarrow (.63, .85)$$

8.80 a. The point estimate for p is $\hat{p} = \frac{x}{n} = \frac{64}{106} = .604$.

b. H_0: $p = .70$
H_a: $p \neq .70$

c. The test statistic is $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{.604 - .70}{\sqrt{\frac{.7(.3)}{106}}} = -2.16$

d. The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the z distribution. From Table IV, Appendix A, $z_{.005} = 2.58$. The rejection region is $z < -2.58$ or $z > 2.58$.

e. In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 106(.70) = 74.2$ and $nq_o = 106(1 - .70) = 106(.30) = 31.8$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.
Since the observed value of the test statistic does not fall in the rejection region ($z = -2.16 \not< -2.58$), H_0 is not rejected. There is insufficient evidence to indicate the proportion of consumers who believe "Made in the USA" means 100% of labor and materials are from the United States is different from .70 at $\alpha = .01$.

8.84 a. $\hat{p} = \frac{x}{n} = \frac{315}{500} = .63$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 500(.60) = 300$ and $nq_o = 500(1 - .60) = 500(.40) = 200$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if the GSR for all scholarship athletes at Division I institutions differs from 60%, we test:

H_0: $p = .60$
H_a: $p \neq .60$

The test statistic is $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}} = \frac{.63 - .60}{\sqrt{\frac{.60(.40)}{500}}} = 1.37$

(continued)

8.84 (continued)

The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the z-distribution. From Table IV, Appendix A, $z_{.005} = 2.575$. The rejection region is $z > 2.575$ or $z < -2.575$.

Since the observed value of the test statistic does not fall in the rejection region ($z = 1.37 \not> 2.575$), H_0 is not rejected. There is insufficient evidence to indicate the GSR for all scholarship athletes at Division I institutions differs from 60% at $\alpha = .01$.

b. $\hat{p} = \frac{x}{n} = \frac{84}{200} = .42$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 200(.58) = 116$ and $nq_o = 200(1 - .58) = 200(.42) = 84$. Since both of these values are greater than 15, the sample size is large enough to use the normal approximation.

To determine if the GSR for all male basketball players at Division I institutions differs from 58%, we test:

H_o: $p = .58$
H_a: $p \neq .58$

The test statistic is $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}} = \frac{.42 - .58}{\sqrt{\frac{.58(.42)}{200}}} = -4.58$

The rejection region requires $\alpha/2 = .01/2 = .005$ in each tail of the z-distribution. From Table IV, Appendix A, $z_{.005} = 2.575$. The rejection region is $z > 2.575$ or $z < -2.575$.

Since the observed value of the test statistic falls in the rejection region ($z = -4.58 < -2.575$), H_0 is rejected. There is sufficient evidence to indicate the GSR for all male basketball players at Division I institutions differs from 58% at $\alpha = .01$.

8.86 Some preliminary calculations are:

$\hat{p} = x/n = 10/35 = .286$

In order for the inference to be valid, the sample size must be large enough. The sample size is large enough if both np_o and nq_o are greater than or equal to 15.

$np_o = 35(1/3) = 11.67$ and $nq_o = 35(1 - 1/3) = 35(2/3) = 23.33$. Since the first value is not greater than 15, the sample size may not be large enough to use the normal approximation.

To determine if the true fraction of sentences with the verb "buy" that are DODs is less than 1/3, we test:

H_0: $p = 1/3$
H_a: $p < 1/3$

The test statistic is $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0 q_0}{n}}} = \dfrac{286 - .333}{\sqrt{\dfrac{.333(.667)}{35}}} = -.59$

The rejection region requires $\alpha = .05$ in the lower tail of the z distribution. From Table IV, Appendix A, $z_{.05} = 1.645$. The rejection region is $z < -1.645$.

Since the observed value of the test statistic does not fall in the rejection region ($z = -.59 \not< -1.645$), H_0 is not rejected. There is insufficient evidence to indicate that the true fraction of sentences with the verb "buy" that are DODs is less than 1/3 at $\alpha = .05$.