UNIVERSITY OF CALGARY

DEPARTMENT OF MATHEMATICS AND STATISTICS

STAT 213 L05 —-Midterm Exam —- Oct. 29, 2009 — Time: 60 min.

SHOW ALL YOUR WORK TO OBTAIN FULL CREDIT. NAME: Solutions

Marks (10)

1. The number of workers on an assembly line varies due to the level of absenteeism on any given day. In a random sample of production output from several days of work, the following data were obtained, where x = number of workers absent from the assembly line and y = number of defects coming off the line.

x	3	5	0	2	1
y	16	20	9	12	10

7 (a) Find the equation of the least-squares line, with x as the independent variable and y as the dependent variable.

x_i	y_i	x_i^2	x_iy_i	y_i^2
3	16	9	48	256
5	20	25	100	400
0	9	0	0	81
2	12	4	24	144
1	10	1	10	100
$\Sigma =$ 11	67	39	182	981

3

$$b = \frac{\Sigma x_iy_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\Sigma x_i^2 - (\Sigma x_i)^2/n}$$

2

$$= \frac{182 - 11(67)/5}{39 - (11)^2/5} = \frac{34.6}{14.8} = \underline{2.338}$$

$$a = \bar{y} - b\bar{x} = \frac{57}{5} - 2.338\frac{11}{5}$$

$$= 13.4 - (2.338)(2.2) = \underline{8.256}$$

So $\boxed{y = 8.256 + 2.338x}$ 2

3 (b) Calculate the correlation coefficient r.

$$r = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\Sigma(x_i-\bar{x})^2}\sqrt{\Sigma(y_i-\bar{y})^2}}$$

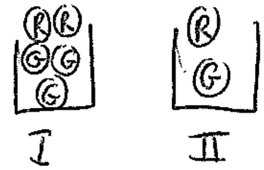
$$= \frac{34.6}{\sqrt{14.8}\sqrt{981 - (67)^2/5}} = \frac{34.6}{\sqrt{14.8}\sqrt{83.2}} = \underline{0.9860}$$

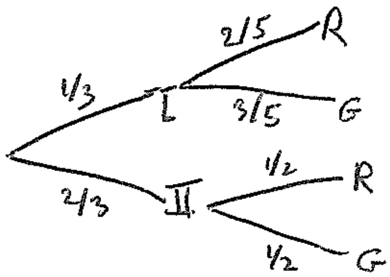
ID number: ____________________

(10) 2. Urn I contains two red balls and three green balls. Urn II contains one red ball and one green ball. A fair die is rolled once. If a 1 or a 2 comes up, then a ball is selected at random from Urn I. Otherwise (if a 3, 4, 5 or 6 comes up) a ball is selected at random from Urn II.

Find:

6 (a) the probability that a green ball is selected;





$$P(G) = P(\mathrm{I} \cap G) + P(\mathrm{II} \cap G)$$

$$= \frac{1}{3}\,\frac{3}{5} + \frac{2}{3}\,\frac{1}{2} = \underline{\frac{8}{15}} = \underline{0.5333}$$

4 (b) the conditional probability that the ball was drawn from Urn II, given that the selected ball was green.

$$P(\mathrm{II} \mid G) = \frac{P(\mathrm{II} \cap G)}{P(G)} = \frac{2/3 \cdot 1/2}{8/15} = \frac{5}{\underline{8}} = \underline{0.625}$$

[Note: partial credit was given for partially right solutions, but no more than 2 points were given for solution in part (a) or part (b) that were not entirely right].

(10) 3. A fair coin is tossed four times. Let A be the event that the number of heads that are obtained exceeds the number of tails that are obtained. Let B be the event that the first two of the four tosses result in heads.

Find:

(3) (a) $P(A)$;

There were $2^4 = 16$ equally likely points in the samplespace

$$P(A) = P\{3 \text{ or } 4 \text{ Heads}\} = P[HHHT, HHTH, HTHH, THHH, HHHH]$$
$$= \underline{5/16} = \underline{0.3125}$$

(3) (b) $P(B|A)$;

$$P(A \cap B) = P[HHHT, HHTH, HHHH] = 3/16,$$

$$\text{so } P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3/16}{5/16} = \underline{\frac{3}{5}} = \underline{0.6}$$

(3) (c) $P(A|B)$.

$$P(B) = \cancel{P[\ldots]}$$
$$= P[HHHH, HHHT, HHTH, HHTT] = 4/16 = 1/4,$$

$$\text{so } P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/16}{4/16} = \underline{\frac{3}{4}} = \underline{0.75}$$

(1) (d) Are A and B independent events? Explain.

No. — either of the following three reasons are ok:

(1) because $P(A|B) \neq P(A)$

(2) because $P(B|A) \neq P(B)$

(3) because $P(A \cap B) \neq P(A)P(B)$

[at most ~~1~~ 1 point of partial credit was given to wrong answers to (a), (b) and (c)].

(10) 4. Sixty percent of the residents in a large city are registered to vote. If 20 residents are selected at random, find:

(3) (a) the probability that exactly 14 will be registered to vote;

Let X = # registered to vote

$X \sim Bin(n=20, p=0.6)$

$P[X=14] = \binom{20}{14}(0.6)^{14}(.4)^{6} = \underline{\underline{0.1244}}$

OR use the Table: $P[X=14] = P[X \le 14] - P[X \le 13]$

$= 0.874 - 0.750 = \underline{\underline{0.124}}$

(3) (b) the probability that at least 11 will be registered to vote.

$P[X \ge 11] = 1 - P[X \le 10] = 1 - 0.245 = \underline{\underline{0.755}}$

Table $n=20, p=.6, k=10$

(4) (c) Let X denote the total number of people, out of the 20 selected, who are registered to vote. Find the mean and variance of X.

$\mu = np = 20(.6) = \underline{\underline{12}} \leftarrow (2)$

$\sigma^2 = np(1-p) = 20(.6)(.4) = \underline{\underline{4.8}} \leftarrow (2)$

[Maximum partial credit for (a) or (b) was 1].

(10) 5. A random variable X has the following probability distribution:

x	$P[X = x]$
0	0.4
1	0.1
2	0.3
3	0.2

5 (a) Compute $P\{(X - 1.5)^2 \geq 2\}$.

x	$p(x)$	$(x-1.5)^2$
0	.4	2.25
1	.1	0.25
2	.3	0.25
3	.2	2.25

$$P[(X-1.5)^2 \geq 2] = P[X=0] + P[X=3]$$
$$= 0.4 + 0.2 = \underline{\underline{0.6}}$$

5 (b) Compute the expected value and standard deviation of X.

$$EX = \sum x\,p(x) = 0(.4) + 1(.1) + 2(.3) + 3(.2)$$
$$= 0 + .1 + .6 + .6 = \underline{\underline{1.3}} \leftarrow 2$$

$$E(X^2) = \sum x^2 p(x) = 0 + .1 + 4(.3) + 9(.2)$$
$$= 0.1 + 1.2 + 1.8 = 3.1$$

$$\text{so } Var(X) = E(X^2) - (EX)^2 = 3.1 - (1.3)^2$$
$$= 1.41,$$

$$\text{so } \sigma = \sqrt{1.41} = \underline{\underline{1.187}} \leftarrow 3$$

50 Total Marks for the Exam.