UNIVERSITY OF CALGARY

DEPARTMENT OF MATHEMATICS AND STATISTICS

STAT 213 L05 B19/B20 —- Quiz no.5 —- Dec. 2, 2009— TIME: 30 minutes

SHOW ALL YOUR WORK TO OBTAIN FULL CREDIT. NAME: Marking Key

Marks 5

1. The moisture content per pound of a dehydrated protein concentrate is known to have a distribution with mean 3.5 and standard deviation 0.5. A random sample of 64 specimens, each consisting of one pound of this concentrate, is taken and the moisture content of each is measured. Find (approximately) the probability that the sample mean of these 64 measurements:

3 (a) will exceed 3.6;

By the Central Limit Theorem, $\bar{X}$ is approximately normally distributed with mean $\mu = 3.5$ and standard deviation $\frac{\sigma}{\sqrt{n}} = \frac{0.5}{\sqrt{64}}$

$= \frac{0.5}{8} = 0.0625$. ← 1

$$P[\bar{X} > 3.6] = P\left[\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} > \frac{3.6-3.5}{.0625}\right] \approx P\left[Z > \frac{.1}{.0625}\right] \leftarrow 1$$

$$= P[Z > 1.6] = .5 - P[0 < Z < 1.6] = .5 - .4452 = 0.0548 \leftarrow 1$$

2 (b) will be between 3.42 and 3.58.

$$P[3.42 < \bar{X} < 3.58] \approx P\left[\frac{3.42-3.5}{.0625} < Z < \frac{3.58-3.42}{.0625}\right] \leftarrow 1$$

$$= P[-1.28 < Z < 1.28] = 2P[0 < Z < 1.28]$$

$$= 2[.3997] = 0.7994 \leftarrow 1$$

ID number: ____________________

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2. A new treatment is investigated to see how much it prolongs the lives of terminal cancer patients. The treatment is administered to 75 randomly-selected patients, and then the duration of their survival times are recorded. The sample mean and sample standard deviation of the 75 survival times are found to be 5.2 years and 1.3 years, respectively. Compute a 95% confidence interval for the mean survival time under the new treatment.

$95 = 100(1-\alpha)$, so $\alpha = .05$, $z_{\alpha/2} = z_{.025} = 1.960$ ← ½

$\bar{x} = 5.2$, $s = 1.3$, $n = 75$

95% C.I. is $\bar{x} \pm z_{\alpha/2} \frac{s}{\sqrt{n}}$ ← ½

or $5.2 \pm 1.96 \frac{1.3}{\sqrt{75}}$ ← ½

or 5.2 ± 0.294 ← ½

or $\boxed{4.906\text{yr} < \mu < 5.494\text{yr}}$ is 95% C.I. for μ.

3

3. Let X_1, X_2 and X_3 be independent random variables, each having the following discrete distribution:

x	$P[X = x]$
1	0.7
2	0.3

Find the sampling distribution of $Y = X_1 + X_2 + X_3$..

y	$P[Y = y]$
3	.343
4	.441
5	.189
6	.027

subtract (1) mark for each incorrect pair $\{y, P[Y=y]\}$.

X_1	X_2	X_3	y	Prob.
1	1	1	3	.343
1	1	2	4	.147
1	2	1	4	.147
1	2	2	5	.063
2	1	1	4	.147
2	1	2	5	.063
2	2	1	5	.063
2	2	2	6	.027
				1.00

10 marks total