Statistics 217 – Review Problems

1. In the past a chemical plant has produced an average of 1100lb of a chemical per day. A random sample of 260 operating days from this year shows that the sample mean and standard deviation are 1040 and 360, respectively. The plant manager wishes to test whether the average daily production has dropped significantly over the past year.

    (a) Give the appropriate null and alternative hypotheses.
    (b) Determine the rejection region corresponding to a level of significance of $\alpha = 0.05$.
    (c) Do the data provide sufficient evidence to indicate a drop in average daily production?
    (d) What should be the smallest level of significance one should select to reject the null hypothesis with this data?

2. Although the Occupational Safety and Health Act (OSHA) is not very popular with management because of the cost of implementing its requirements, some sources claim that it has been effective in reducing industrial accidents. The data in the following table were collected on lost-time accidents (the figures given are mean man-hours lost per month over a period of one year), both before and after OSHA came into effect. Data were recorded for 7 industrial plants.

Period	Plant Number						
	1	2	3	4	5	6	7
Before OSHA	38	64	42	70	58	30	31
After OSHA	31	58	43	65	52	29	28

    (a) Assume lost man-hours are normally distributed.
        (i)   Do these data provide sufficient evidence to indicate that OSHA has been effective in reducing lost-time accidents? Use $\alpha = 0.1$.
        (ii)   Determine the p-value of the test. Compare this with α used in (i)
    (b) Suppose that we cannot assume that lost man-hours are normally distributed. Carry out an appropriate statistical test procedure to answer the question in (a) (i).

3. There are many "indicators" that investors use to predict the behavior of the stock market. One of these is the "January indicator". Some investors believe that if the market is up in January, then it will be up for the rest of the year. On the other hand, if it is down in January, then it will be down for the rest of the year. The following table gives the data for the 72 years from 1916 to 1987:

February-December	January	
	Up	Down
Up	33	13
Down	13	13

    (a) At the 5% significance level, is there sufficient evidence to conclude that the January indicator is an effective way to predict whether the market will be up or down?
    (b) Determine the p-value.

4. Are subcompacts noticeably less expensive to operate than compact cars? To explore this issue, a private testing agency withes to estimate the difference in the average operating cost per mile between subcompacts and compacts, accurate to within 1 cent per mile, with a confidence of approximately 95%. The agency knows from past experience that the standard deviation of total operating costs per mile (including depreciation, maintenance, gas and oil, insurance, and taxes) is about 3 cents for each type of car.

    If the agency is willing to use an equal number of cars in each group, how many cars should the testing agency include in each group?

5. A student wanted to know whether the answers a, b, c, d and e on the standardized departmental multiple choice test occurred equally as often. Several old departmental tests were randomly selected and the occurrences of each answer was tabulated.

Answer	Rimes Used as Answer
A	39
B	26
C	43
D	42
E	25

Test whether all answer choices are equally likely or not. Use a significance level of 0.05.

6. A shoe manufacturer wanted to test whether there is a difference in the amount of wear on three different designs of rubber soles for a particular jogging shoe. Nine joggers were selected for the experiment. Each type of design was randomly assigned to three joggers.   After running 200 miles, the joggers turned in their shoes. The manufacturer used an index to indicate the amount of rubber left on the sole. The measurements obtained were:

Design 1:	3.2	4.1	6.2
Design 2:	4.7	6.1	4.0
Design 3:	5.0	6.0	5.7

Assume that the measurements for each design follow a normal distribution with equal variances. Is there sufficient evidence, at the 5% significance level, to conclude that the designs are different with regard to average wear?

7. All job applicants for sales positions in a large company are administered a screening test. Scores on this test are used to predict potential future volume of sales of individual applicants. Following are the test scores and first month volume of sales for several recently hired salesmen:

Test Score	Sales (in $1,000)
17	21
11	13
21	23
9	12
14	15

Computer output could be given for this problem. The output for this problem follows. Use manual calculations and the computer output to answer the questions to give yourself practice doing the work both ways.

Linear Regression

$\sum x = 72.0$ $\qquad$ $\sum y = 84$

$\sum x^2 = 1128.0$ $\qquad$ $\sum y^2 = 1508.0$

$\sum xy = 1301.0$ $\qquad$ n=5

Parameter	Value	St.Dev	T-ratio
Intercept	2.368421	2.070594	1.143837
Slope	1.002193	0.137856	7.269860

S=1.316506 $\qquad$ Rsquared= 0.946286

ANOVA TABLE

Source	DF	SS	MS	F
Regression	1	91.600439	91.600439	52.850865
Residual	3	5.199561	1.733187	
Total	4	96.800		

(a) Find the regression equation of volume of sales on the screening test score.

(b) Estimate the standard error of the regression.
(c) Test the significance of regression (non-zero slope) using the t-test ($\alpha = 0.05$).
(d) Test the significance of regression (non-zero slope) using the F-test (($\alpha = 0.05$).
(e) Find the coefficient of determination and the coefficient of correlation. Interpret these values.
(f) Find the 99% confidence interval for the average monthly sales of all salesmen scoring 15 points on the screening test.
(g) Mr. Longshot just completed the test and scored 15 points. Find the 99% confidence interval for his actual first month sales.

8. Alberta Hotel and Restaurant Association has decided to investigate spending habits of customers in its member restaurant establishments. One of the problems is to find whether the amounts spent for a dinner by a man in a "wining and dining" couple is the same as the amount spent by a woman. It was observed that most couples spend a moderate amount of money but that there are a few "high rollers" who spend a small fortune to entertain and hence the distribution for the dinner bills as well as the distribution of the size of difference between the cost of the man's and the woman's dinner are not normally distributed. Following are spending records of several couples:

Couple #	Man ($)	Woman ($)
1	21.10	19.3
2	31.10	35.7
3	51.3	79.9
4	25.6	24.3
5	28.8	29.20
6	75.2	94.3
7	23.6	26.8

Do the data indicate that men and women do not spend the same amount on dinner? Assume $\alpha = 0.05$.

9. A high fashion boutique chain has three different retail outlets in a large metropolitan area. It is expected that the store location may have a significant effect on the amounts spent by individual customers and hence on the volume of sales. For this reason several sales were randomly chosen from each outlet for comparisons. The manager knows that even though most sales are within a specific price range, some high ticket sales make the distribution of sales highly skewed. Following are the amounts (in $) of the selected bills:

Store #1	Store #2	Store #3
23	38	43
31	24	37
18	36	29
25	31	75
15	26	39

Is there a significant difference between the retail outlet locations with regards to the average amount spent by individual customers? Use an appropriate test with $\alpha = 0.05$.

10. A photo lab advertises that it takes no more than 1.5 hours (on average) to process a customer's film. Assume processing time has a standard deviation of 2 hours.
    (a) A consumer group does not believe this claim but is prepared to accept it if the average processing time of 100 films is no more than 1.91 hours
        (i)   What is the probability of a Type I error in this case?
        (ii)   Using the test procedure outlined above, what is the probability that the consumer group will accept the lab's claim when, in fact, the (population) mean processing time is 1.7 hours?
    (b) A consumer group estimates the average processing time to be 1.76 hours. They state that they are 98% sure that this estimate is in error by no more than 0.233 hours.

(i) How large of a sample was used?
(iii)   Do these results indicate that the advertisement is not true? Why or why not?

11. A survey is to be carried out to determine if the majority (more than 50%) of students applying to enter the Faculty of Management have completed the pre-requisite courses within a two-year period.
    (a) State the hypothesis to be used in testing.
    (b) Suppose a random sample of 400 applicants is to be used.
        (i)   What is the minimum number of applicants in the sample that have completed the courses within two years to conclude that the majority of applicants have done so? Assume $\alpha = 0.05$.
        (ii)   Suppose 210 students in the sample of 400 have completed the courses within 2 years. Determine the p-value (attained significance level) of the test.

12. A study was carried out to see if a new paint would reduce corrosion of car bodies. The following data are results of testing the new paint and the standard paint under similar conditions. The data represents the amount of rusting of each sample (the smaller the number, the less the amount of rust).

New Paint:	8	7	11	8	9	8
Standard Paint:	21	24	27	19	24	

    (a) Testing at the 1% significance level, can we conclude that the new paint is better (reduces corrosion)?
        (i)   Assume that the populations are normally distributed with the same variance.
        (ii)   Assume that the populations are normally distributed. (No equal variance assumption).
    (b) Carry out a non-parametric test procedure to answer the question posed in (a). Use $\alpha = 0.05$.

13. The amount of time people take to respond to driving hazards varies from one individual to another. An automobile club tested the response time of a random sample of 30 people and found the average response time to be 3.4 seconds with a standard deviation of 0.37 seconds. Assume response times are normally distributed.
    (a) The automobile club believes the response time should not exceed 3 seconds. Do the data indicate that the response time may exceed 3 seconds? Test at the 1% significance level.
    (b) Determine the p-value of the test in (a). Compare this with the significance level used in (a).
    (c) Someone stated that they thought that the standard deviation was significantly less than 0.5 seconds. Testing at the 5% significance level, do the data support this belief?
    (d) Determine the p-value of the test in (c).

14. In the summary of a survey of smoking habits of North American adults, it was stated that 25.2 % smoke now as compared to 54.7% in 1982. The results were based on a random sample of 100 adults in each year.
    (a) Determine a 95% confidence interval estimate of the difference in the percentage of North American adults that smoked in 1982 and now.
    (b) Do the data indicate that there has been a reduction in the percentage of smokers in North America over this time period? Use the results of (a) to answer this and explain why you have come to your conclusion.
    (c) State the hypothesis of (b).

15. A manufacturer wants to determine if there is any difference between 3 processes that are used to make the same items. Weight of the items is the critical variable in studying the difference between the processes. Sample are taken from each process's production at five randomly selected times and the weights of the items are recorded for each sample. The data (weights) were summarized in a table where the rows represent the different processes and the columns represent the times. Following is a partially complete analysis of variance table based on the data:

Two-Way ANOVA Table

Source	SS	DF	MS	F
TRT (process)	45.6			
Block (time)			15.0	
Error				
Total	136.0			

(a) Complete the analysis of variance table.
(b) Do a complete analysis of the data using a 5% significance level for testing.

16. The makers of Slim, a diet cola, advertise the product to contain no more than 15 calories per bottle. In response to complaints by nutrition experts, the company has a random sample of bottles analyzed, giving the following amounts of calories per bottle:

   14.9   14.8   15.7   15.9   16.3   17.0   16.5   15.0   15.7   16.7   14.9   17.3

It is assumed that the distribution of the number of calories is normal with a standard deviation of 0.9. Based on the data, what conclusions can be made regarding the advertisement?

17. A machine makes metal plates that are used in a product. The diameter of these plates is approximately normally distributed with a mean of 5 mm. As long as the standard deviation of the diameters does not exceed 0.95 mm the production process is considered to be under control and the plates will be acceptable. However, if the standard deviation of the diameter of the plates is significantly greater than 0.95, the machine must be adjusted.

   A random sample of 11 metal plates had a standard deviation of 1.27 mm. Does this indicate that the machine should be adjusted? Test the appropriate hypothesis at the 5% significance level.

18. A law firm claims that the average cost per liability suit does not exceed $100,000. For a random sample of 16 of the firm's cases the average settlement was $107,000 with a standard deviation of $15,000. Assume settlement amounts are approximately normally distributed.

   (a) Testing at the 5% significance level, do the data indicate that the firm's claim is false?
   (b) Determine the p-value for the test carried out in (a) and compare this with the value of α used in (a).

19. A human resources manager want to compare the average number of days missed on "sick leave" per year between female and male employees. Independent sample of records of female and male employees gave the following summary data on the number of "sick leave" days:

Employees	Female	Male
Sample size	100	50
Mean	14.8 days	10.2 days
Standard Deviation	0.8 days	1.6 days

   (a) Determine a 95% confidence interval estimate of the difference in the average "sick leave" days for the male and female employees.
   (b) Do the results of (a) indicate that there is any difference in the average number of "sick leave" days per year for male and female employees? Explain why. No calculations required.

20. A machine produces metal rods that should be at least 507.5 cm in length. Some variability in length is expected in the individual rods produced; but if the length is less than 507..5 cm on average, the machine should be stopped and adjusted.

A random sample of rods is taken from the day shift's production of the machine, the lengths of which are given below. We want to know if we should tell the shop foreman to stop the machine and adjust it based on the sample.

Length of rods in centimeters: 505   507   502   504   508   509   508   506   505

(a) Assume that the length of rods is normally distributed.
       (i)   Set up the appropriate hypothesis.
       (ii)   Determine the p-value for the test.
       (iii)   Assuming testing is carried out at the 5% significance level, what would you advise the shop foreman" Use the results of (b) to reach your conclusion. Explain why. No calculations.
(b) Carry out a non-parametric test procedure to answer this. Assume that the probability of making a Type I error is 0.05.