

STAT 217
Assignment #3
Chi-squared, F-test, and two sample hypothesis testing

The directions for the chi-square and F distribution for MINITAB are the same as the directions for the standard normal. The only difference is that you have to also plug in degrees of freedom (like the t distribution). Practice using the tables and the computer.

1. Some of these are best found using the computer.

- (a) $P(\chi^2 \leq 19.02)$ $df = 9$ (.975)
 (b) $P(\chi^2 \leq 1.44)$ $df = 6$ (0.0366)
 (c) $P(\chi^2 \geq 10.52)$ $df = 20$ (.9577)
 (d) $P(4.33 \leq \chi^2 \leq 12.2)$ $df = 15$ (.3326)

2. Determine the value of χ^2_o if it is known that:

- (a) $P(\chi^2 \geq \chi^2_o) = 0.20$ $df = 25$ (30.6752)
 (b) $P(5.66 \leq \chi^2 \leq \chi^2_o) = 0.25$ $df = 12$ (9.2412)
 (c) $P(\chi^2 \leq \chi^2_o) = 0.40$ $df = 15$ (13.0297)

3. (a) The listed values are waiting times (in minutes) of customers at the Humber Valley Credit Union, where customers enter a single waiting line that feeds three windows. Construct a 95% confidence interval for the population standard deviation. [$s^2 = .2272, (.3279, .8703)$]

6.5 6.6 6.7 6.8 7.1 7.3 7.4 7.7 7.7 7.7

- (b) The listed values are waiting time (in minutes) of customers at the Durham Credit Union, where customers may enter any one of three different lines that have formed at three different teller windows. Construct a 95% confidence interval for the population standard deviation. [$s^2 = 3.3182, (1.253, 3.3255)$]

4.2 5.4 5.8 6.2 6.7 7.7 7.7 8.5 9.3 10.0

- (c) Interpret the results found in (a) and (b). Do the confidence intervals suggest a difference in the variation among waiting times? Which arrangement seems better: the single-line system or the multiple-line system with respect to variability? [CIs do not overlap meaning that they most likely have different population standard deviations]

4. Suppose that a machine dispenses sand into bags. A sample of 100 bags is taken from a new machine and the standard deviation is 14kg. The population standard deviation is known to be 18kg.

- (a) Test at the 5% significance level that the new machine is better. What assumptions were made? [$H_0: \sigma = 18\text{kg}$, $H_a: \sigma < 18\text{kg}$, $\chi^2_{\text{crit}} = 77.0463$, $\chi^2_{\text{calc}} = 59.8889$, RHo, random sample, normal population]
 (b) What's the critical value for the sample standard deviation in (a)? $s_c = 15.8793$

5. Last year, the mean number of books borrowed per cardholder at a major university was 18.2 books per semester with a st.deviation of 4.2. A random sample of 25 cardholders showed the following results for this semester: $s^2 = 6.17$. The library administration would like to know whether this semester's variance is the same as last semester's at the 1% significance level. What assumptions were made? [$H_0: \sigma = 4.2$, $H_a: \sigma \neq 4.2$, $\chi^2_{\text{crit}} = 9.88623, 45.5585$, $\chi^2_{\text{calc}} = 8.3946$, RHo]

6. A sample of 41 skulls is obtained from Egyptian males who lived around 1850 B.C. The maximum breadth of each skull is measured with the results that the mean is 134.5 mm and the standard deviation is 3.5 mm.

- (a) Using these sample results, construct a 95% confidence interval for the population standard deviation. [2.8735, 4.4783]
 (b) Past studies have shown that the mean is 134.5 with standard deviation of 2.5 for Egyptian males who lived around 1850 B.C. Does the sample support this? Explain the new findings with respect to the old ones.
 (c) Set up the hypothesis test to see if the new standard deviation is different from the one of 2.5 at the 5% significance level. [$H_0: \sigma = 2.5$, $H_a: \sigma \neq 2.5$, $\chi^2_{\text{crit}} = 24.4331, 59.3417$, $\chi^2_{\text{calc}} = 78.4$, RHo]

7. Some of these are best found using the computer.

- (a) $P(F \leq 4.77)$ $df = 9,5$ (.95)
 (b) $P(F \leq 1.44)$ $df = 6,7$ (0.6799)
 (c) $P(F \geq 3.52)$ $df = 20,8$ (.0365)
 (d) $P(1.33 \leq F \leq 2.56)$ $df = 15,11$ (.2604)

8. Determine the value of F_{α} if it is known that:

- (a) $P(F \geq F_{\alpha}) = 0.13$ $df = 25,18$ (1.67)
 (b) $P(1.66 \leq F \leq F_{\alpha}) = 0.14$ $df = 12,9$ (2.52)
 (c) $P(F \leq F_{\alpha}) = 0.624$ $df = 5,7$ (1.26)

You can use the computer to test two population variances, means and proportions. Go to Stat> Basic statistics> and then pick the test that you want it to do. The rest is fairly straight forward. When the population variances are not equal and the sample sizes are small, the computer is especially handy when finding the degrees of freedom instead of calculating it by hand. On a test, you are still expected to show work even if you use the computer.

9. Business schools A and B reported the following summary of GMAT verbal scores:

	N	\bar{x}	s^2
A	11	34.75	48.59
B	13	33.74	44.68

- (a) At the 5% significance level, is the variance of school A greater than school B? [$H_0: \sigma_A^2 = \sigma_B^2$,
 $H_a: \sigma_A^2 > \sigma_B^2$ $F_{crit} = 2.75$, $F_{calc} = 1.0875$, Fail to RHO, this could also be set up as a left-tailed test
 $H_0: \sigma_B^2 = \sigma_A^2$, $H_a: \sigma_B^2 < \sigma_A^2$ $F_{crit} = .3436$, $F_{calc} = .9195$, Fail to RHO.]
- (b) Construct a 95% confidence interval for the true population standard deviation ratio. Interpret. [(0.5681, 1.9841) or (.5040, 1.7605)]

10. 16 alkaline and 16 heavy-duty Radio shack batteries were placed individually in a circuit consisting of two flashlight bulbs wired in parallel, a switch, a battery holder, and a Hewlet Packard 427-A analog DC voltmeter. Each battery was drained to a reference failure voltage of 0.9 volts and the time to failure (min) was measured.:

	Alkaline				Heavy-duty			
15	31	17	18	29	22	22	27	
30	33	18	15	26	17	22	23	
16	29	26	24	23	27	23	24	
30	19	22	20	22	25	22	26	

Assume the populations of times to failure is normally distributed.

- (a) Test the hypothesis at the 0.05 level that the two types of batteries have the same population variances. [$H_0: \sigma_A^2 = \sigma_H^2$, $H_a: \sigma_A^2 \neq \sigma_H^2$ $F_{crit} = .3497$, 2.86, $F_{calc} = 4.8409$, Rho could have reversed
 $H_0: \sigma_H^2 = \sigma_A^2$, $H_a: \sigma_H^2 \neq \sigma_A^2$ $F_{crit} = .2066$, Rho]
- (b) Find the p-value. [.0042]
- (c) Find the 95% confidence interval for the ratio of standard deviations. [(1.3010, 3.7206) or (.2688, .7686)]
- (d) Using the results in either (a) or (c), test to see if the means are equal at the 5% significance level. What assumptions did you make? [$H_0: \mu_A = \mu_H$, $H_a: \mu_A \neq \mu_H$ $df = 20$, $t_{crit} = \pm 2.086$, $t_{calc} = -1.6127$, fail to RHO, or $H_0: \mu_H = \mu_A$, $H_a: \mu_H \neq \mu_A$ $t_{calc} = .6127$]

11. Two marathon training procedures are tried for comparison purposes. Their efficacy is to be determined in a marathon race. Assume race times are normally distributed and variances are not equal. The following results were observed:

	No. In sample	mean race time	s.d of race times
Procedure 1	15	150 min	12 min
Procedure 2	26	170 min	15 min

- (a) Test that procedure 2 is greater than procedure 1 at $\alpha = 0.025$ [$H_0: \mu_2 = \mu_1$, $H_a: \mu_2 > \mu_1$ $t_{crit} = 2.032$, $df = 34$, $t_{calc} = 4.6812$, Rho, we could have also set this up as a left tailed test]
- (b) Find a 90% confidence interval for the difference in the mean race times. [$t = 1.69092$, (12.7756, 27.2244) or (-27.2244, -12.7756)]
- (c) Find a 90% confidence interval for the difference in the mean race times assuming population variances are the same. [$t = 1.6849$ $sp = 13.9973$, (12.3532, 27.6468) or (-27.6468, -12.3532)]

12. A large brokerage house wants to see if the percent of new accounts valued at over \$50,000 has changed over the year. A random sample of 900 accounts opened last year showed that 27 were over \$50,000 in size. A random sample of 1,000 accounts opened this year showed 44 that were in excess of \$50,000. Does that data indicate that there has been a change in the percent of new accounts valued at over \$50,000?

- (a) Carry out the appropriate statistical test at a 1% significance level. [$H_0: p_1 = p_2$, $H_a: p_1 \neq p_2$ $z_{crit} = \pm 2.575$, $z_{calc} = -1.607$, Fail to RHo or $H_0: p_2 = p_1$, $H_a: p_2 \neq p_1$ $z_{crit} = \pm 2.575$, $z_{calc} = 1.607$, Fail to RHo]
- (b) Determine the minimum level of significance to conclude that the data indicates that there has been a change. [p-value=0.1074]

13. The occupational Health and Safety Act is not popular with Management because of the cost of implementing its requirements. However, some sources claim it has been effective in reducing industrial accidents. Listed are the man-hours lost due to accidents at 6 randomly selected plants.

Plant number	1	2	3	4	5	6
Before OHSA	38	64	42	70	58	30
After OHSA	31	58	43	65	52	29

- (a) Is there sufficient evidence to indicate that The Occupational Health and Safety Act has been effective in reducing lost-time accidents? Test at $\alpha = 0.10$. [$H_0: \mu_d = 0$, $H_a: \mu_d > 0$ (this is where $d = \text{before} - \text{after}$ $\bar{d} = 4$ $sd = 3.2249$, $t_{crit} = 1.476$, $t_{calc} = 3.038$, RHo, or you could have set this up as a left tailed test where $d = \text{after} - \text{before}$]
- (b) Find the P-value in (a). [0.0144]
- (c) What assumptions did you make in (a)?

14. Let p_1 denote the percentage of people who were unemployed in March, and let p_2 denote the percentage of people who were unemployed in August. Suppose that during late March, the government instituted policies designed to lower the unemployment rate. We want to test whether the policies were effective at $\alpha = 0.05$. In March, in a random sample of 1000 people, 75 were unemployed. During August, in an independent random sample of 1000 people, 65 were unemployed. What is your conclusion? [$H_0: p_1 = p_2$, $H_a: p_1 > p_2$ $z_{crit} = 1.645$, $z_{calc} = 0.8765$, Fail to RHo or we could have set this up as a left tailed test]

15. A producer of barley wanted to know which of two fertilizers, A or B, was better for growing barley. To study this, seven plots of land were selected. Each plot was divided in half. For each plot, fertilizer A was assigned to one half and fertilizer B to the other half using the toss of a coin. The yield of barley in bushels per acre after a period of time gave the following data:

Plot	1	2	3	4	5	6	7
A	42	38	41	43	42	38	41
B	40	42	36	39	41	40	36

Does the above data indicate one of the fertilizers is more effective than the other? Use $\alpha = 0.05$

[Ho: $\mu_d = 0$, Ha: $\mu_d > 0$ (this where $d=A-B$ $\bar{d} = 1.5714$ $sd = 3.5051$, $t_{crit} = 1.943$, $t_{calc} = 1.186$, Fail to RHo, this could be set up as a left tailed test where $d=B-A$]

16. A USA Today study reported the longest average workweeks for non-supervisory employees in private industry to be mining (45.4 hours) and manufacturing (42.3 hours). The same article reported the shortest average workweeks to be retail trade (29 hours) and services (32.3 hours). A study conducted in the state of Illinois found the following results:

Industry	N	Ave hours per week	Standard deviation
Mining	15	47.3	5.5
Manufacturing	10	43.5	4.9

Assuming that there is a statistically significant difference in the variability of hours per week worked in the mining and manufacturing industries

- Construct a 95% confidence interval estimate of the mean difference in hours per week worked for non-supervisory employees. Interpret this interval. [df = 20, (-.5843, 8.8143) or (-8.8143, .5843)]
- Does the Illinois data suggest that non-supervisory employees in the mining industry, on average, work more hours per week than their manufacturing industry counterparts? Regulate the probability of type I error at 5 percent. [Ho: $\mu_1 = \mu_2$, Ha: $\mu_1 > \mu_2$ (1: mining 2: manufacturing) $t_{crit} = 1.725$, $df = 20$, $t_{calc} = 1.808$. RHo this could also be set up as a left tailed test]
- What is the p-value of your results in (b)? [0.0428]

17. Of a random sample of 100 stocks on the Toronto Stock Exchange, 32 made a gain today. A random sample of 100 stocks on the Montreal Stock Exchange showed 27 stocks making a gain.

- Construct a 95% confidence interval, estimating the difference in the proportion of stocks making a gain on the two exchanges. [(-0.0762, 0.1762) or (-.1762, .0762)]
- Does the data suggest there is a statistically significant difference between the proportion of stocks making gains on the two exchanges? Use $\alpha = 0.05$. [Ho: $p_1 = p_2$, Ha: $p_1 \neq p_2$ $z_{crit} = \pm 1.96$, $z_{calc} = 0.7753$, Fail to RHo or Ho: $p_2 = p_1$, Ha: $p_2 \neq p_1$ $z_{calc} = -.7753$]

18. The residents of Cardston have complained for many years that they pay higher traffic fines than the residents of their adversary, Raymond. A local newspaper hired a statistician to determine whether there was any merit to this argument. The statistician randomly selected 25 residents from Cardston and 25 residents from Raymond, all of whom had received a speeding ticket. Fines are normally distributed and assume that the variances are not equal. The fine paid by each Cardston and Raymond resident were recorded. The summary statistics are as follows:

Town	Mean	Standard deviation
Cardston	\$60.00	\$12
Raymond	\$50.00	\$10

- Construct a 95% confidence interval, estimating the mean difference in traffic fines paid by Cardston residents and Raymond residents. [df = 46, (3.7115, 16.2885) or (-16.2885, 3.7115)]
- Using the p-value approach, does the data provide sufficient evidence to suggest that Cardston residents pay more, on average, in traffic fines than Raymond residents? Use $\alpha = 0.05$. [Ho: $\mu_C = \mu_R$, Ha: $\mu_C > \mu_R$ $df = 46$, $t_{calc} = 3.2009$, 0.00121 , RHo this could also be set up as a left tailed test]

19. A researcher wanted to know if the percentage of drinking drivers increased during later evening hours. She decided to compare the percentage of drinking drivers from 7:00 to 9:00 pm with the percentage from 10:00 to 12:00am. The randomly selected 250 drivers from the first time period and

200 from the second time period. Each driver in both samples was given a breath test and assigned to the category drinking driver or non-drinking driver. From the early period, 12 percent were drinking drivers, and 19 percent from the later period. Does this data suggest that there is a statistically significant increase in the percentage of drinking drivers in the later time period? Use $\alpha = 0.05$. Determine the p-value of your results. How convincing (or unconvincing) is the data?

[Ho: $p_{Early} = p_{Later}$, Ha: $p_{Early} < p_{Later}$ zcrit = -1.645, zcalc = -2.060, RHo, p-value = 0.0197 this could also be set up as a right tailed test]

20. In a province wide poll of 2000 men and 2010 women, 980 and 1025 women report that they are opposed to the death penalty in ALL circumstances.
- (a) Find a 90% confidence interval estimate for the difference in the proportion of men opposed to the death penalty and corresponding proportion of women. [(-0.046, 0.006) or (-.006, .046)]
- (b) Test the claim at a level of significance of 5% that women are less likely to favour the death penalty than men. What is your conclusion? [Ho: $p_F = p_{Male}$, Ha: $p_F > p_{Male}$ zcrit = 1.645, zcalc = 1.2665, Fail to RHo this could also be set up as a left tailed test]
21. Independent random samples taken at two local liquor stores provide the following information regarding customers' purchases.

Store	Bob's Booze	Len's Liquor Emporium
N	46	39
Mean	\$52.40	\$61.75
s ²	9.50	7.25

Does the data indicate that the average purchases at Len's exceed those at Bob's by more than \$5, on average? Use $\alpha = 0.05$. [Ho: $\sigma_B^2 = \sigma_L^2$, Ha: $\sigma_B^2 \neq \sigma_L^2$ df 45,38 fcrit = .5428, 1.8727, fcalc = 1.31, Fail to RHo this could also be set up the other way as Ho: $\sigma_L^2 = \sigma_B^2$, Ha: $\sigma_L^2 \neq \sigma_B^2$ df 38,45 fcrit = .5334, 1.8422, fcalc = .7632, Fail to RHo (assume equal variances) Ho: $\mu_L - \mu_B = 5$, Ha: $\mu_L - \mu_B > 5$ sp = 2.9103 tcrit = 1.645, tcalc = 6.8668, df = 83, RHo this could be set up as a left tailed test]