

Stat 217 Review Solutions

1. (a) $H_0: \mu = 1100$
 $H_a: \mu < 1100$
 - (b) RHo if $Z_{\text{calc}} < -1.645$ (you can use Z because $df=259$)
 - (c) $Z_{\text{calc}} = -2.69$
Dec: RHo, $-2.69 < -1.645$
Conclusion: At the 5% significance level, there's a drop in the average daily production.
 - (d) P-value = $P(z < -2.69) = .0036$
 - (e) $P(z < -2.24) = .0125$ the significance level is $\alpha = .0125$ Since $1040 < 1050$, then we RHo and conclude that there's a drop in the average daily production.

2. (a) (i) $H_0: \mu_d = 0$
 $H_a: \mu_d > 0$ (B - A)
RHo if $t_{\text{calc}} > 2.447$ ($df = 6, \alpha = .025$)
 $t_{\text{calc}} = 3.4382$ ($\bar{d} = 3.8571, s_d = 2.9681$)
Dec: RHo, $3.4382 > 2.447$
Conclusion: At the 2.5% significance level, OSHA has been effective in reducing lost time accidents
Note: you could have done a left tailed test, then you would make all the values negative.
(ii) P-value = $P(t > 3.4382) = .0064$ (computer)
 $0.005 < \text{p-value} < 0.01$ (tables)
 - (b) Wilcoxon Signed-Rank test
 H_0 : the distributions are the same for accidents before and after OSHA
 H_a : There are more accidents before OSHA than after
 $W_+ = 26.5$ $W_- = 1.5$ (B-A)
RHo is $W \leq 2$
Dec: RHo, $1.5 < 2$
Conclusion: At the 2.5% significance level, same as above

3. (a) H_0 : January indicator is independent of market prices the rest of the year
 H_a : January indicator is not independent of market prices the rest of the year (it can be used to predict the market prices for the rest of the year)
RHo is $\chi^2_{\text{calc}} > 3.841$ ($df = 1, \alpha = .05$)
 $\chi^2_{\text{calc}} = 3.381$
Dec: Fail to RHo $3.381 < 3.841$
Conclusion: At the 5% significance level, January indicator is independent of market prices for the rest of the year. It can not be used to predict the market prices for the rest of the year.

Could also do a two population proportions test.

Rho if $|z_{\text{calc}}| > 1.96$, $p^{\wedge}\text{pooled} = .6389$, $P^{\wedge}1 = 33/46 = .7174$, $p^{\wedge}2 = 13/36 = .5$,
 $Z_{\text{calc}} = 1.8447$

Dec: Fail to Rho $1.8447 < 1.96$ and > -1.96 .
Conclusion: same as above.

(b) p-value = $P(\chi^2 > 3.381) = .0660$ (computer)
 $.05 < \text{p-value} < .10$ (tables)
or Proportions: P-value = $2 \times P(z > 1.8447) = .065$

4. $n_s = n_c = n$, $z_{\alpha/2} = 1.96$, error = 1 $\sigma_s^2 = \sigma_c^2 = 9$

$$\text{error} = z_{\alpha/2} \sqrt{\frac{\sigma_s^2}{n_s} + \frac{\sigma_c^2}{n_c}} \quad 1 = 1.96 \sqrt{\frac{9}{n} + \frac{9}{n}}$$

$$n = 69.1488 \sim 70$$

5. Ho: All choices are equally likely
Ha: Not all choices are equally likely
RHo is $\chi^2_{\text{calc}} > 9.48773$ (df = 4, $\alpha = .05$)
 $\chi^2_{\text{calc}} = 8.857$
Dec: Fail to RHo, $8.857 < 9.48773$
Conclusion: At the 5% significance level, there is no indication that all answers are not equally likely.

6. a. Ho: $\mu = 3.2$
Ha: $\mu \neq 3.2$
Rho if $t_{\text{calc}} > 2.306$, or < -2.306
 $t_{\text{calc}} = 2.8535$
Dec: Rho $2.8535 > 2.306$
Conclusion: At the 5% sig. level, the sample data does not support their belief of 3.2.

b. 2.742 or 3.658

c. First test the variances

$$\text{Ho: } \sigma_3 = \sigma_2$$

$$\text{Ha: } \sigma_3 \neq \sigma_2$$

Rho if $F_{\text{calc}} > 4.43$ or $< .2257$

$F_{\text{calc}} = 3.1345$ (or $.3190$ if σ_3 and σ_2 were reversed in Ho and Ha)

Dec: Fail to RHo $3.1345 < 4.43$ and $> .2257$

Conc: At the 5% significance level, the variances are the same.

Assumptions : equal population variances (F-test showed this above)

Independent random samples (9 were randomly assigned to design 2 and 9 were randomly assigned to design 3)

Normal populations (it says to assume this)

$$\text{Ho: } \mu_2 = \mu_3$$

$$\text{Ha: } \mu_2 \neq \mu_3$$

Rho if $t_{\text{calc}} > 2.12$ or < -2.12

$S_p = .544$ $t_{\text{calc}} = -2.076$

Dec: Fail to Rho $-2.076 > -2.12$ and < 2.12

Conc: At the 5% significance level, there appears to be no significant difference between design 2 and 3 with respect to wear.

d.

1 way ANOVA				
Source	SS	DF	MS	F
Treatment	6.2607	2	3.1.037	9.886
Error	7.5911	24	0.31629	
Total	13.8518	26		

Ho: Designs are the same with regard to wear

Ha: Designs are not the same with regard to wear

Rho if $F_{calc} > 3.40$ ($df= 2,24 \alpha = .05$) (one-way ANOVA)

$F_{calc} = 9.886$

Dec: RHo, $9.886 > 3.4$

Conclusion: at the 5% significance level, not all designs have the same wear.

Assume normal populations, independent random samples, equal population variances.

(b) a. Ho: median = 3.2

Ha: median $\neq 3.2$

$N=8 T+=33 T-=3 T=3$

Rho if $T \leq 4$

Dec: Rho $3 < 4$

Conc: At the 5% significance level, the median is not 3.2

c. Ho: Design 2 is the same as design 3

Ha: Design 2 is not the same as design 3

$T1=65$ (design 2) $T2=106$ (design 3) $n1=8, n2=8$ Rho is $T \leq 49$ or ≥ 87

$T = 65$ (or 106 since sample sizes are the same)

Dec Fail to Rho $65 > 49$ and < 87

Conc: At the 5% significance level, there is no significant difference in design 2 and 3 with respect to wear.

d. Ho: all 3 designs have the same wear

Ha: not all designs have the same wear

Rho if the test statistic > 5.99147

$$KW = \frac{12}{27(28)} \left(\frac{68^2}{9} + \frac{129.5^2}{9} + \frac{180.5^2}{9} \right) - 3(28) = 11.1931$$

Dec: Rho $11.1931 > 5.99147$

Concl: At the 5% significance level, not all designs have the same wear.

7.

Parameter	Value	St.Dev	T-ratio
Intercept	2.368421	2.070594	1.143837
Slope	1.002193	0.137856	7.269860

$$S = 1.316506 \left(\sqrt{MSE} \right) \quad \text{Rsquared} = 0.946286 \text{ (SSR/SST)}$$

ANOVA TABLE				
Source	DF	SS	MS	F

Regression	1	91.600439	91.600439	52.850865
Residual	3	5.199561	1.733187	
Total	4	96.800		

(b) $\hat{y} = 2.368421 + 1.002193(x)$

(c) $s = 1.316506$

(d) $H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

RHo if $t_{calc} < -3.182$ or > 3.182 ($df=3, \alpha = .05$)

$T_{calc} = 1.002193 - 0 / 0.1379856 = 7.27$

Dec: RHo, $7.27 > 3.182$

Conclusion: At the 5% significance level, there is a linear association between sales and test score

(e) $H_0: \beta_1 = 0$

$H_a: \beta_1 \neq 0$

RHo if $F_{calc} > 10.12$ ($df=1,3 \alpha = .05$)

$F_{calc} = 52.851$ ($\sim 7.27^2 = t_{calc}^2$)

Dec: RHo, $52.851 > 10.12$

Conclusion: At the 5% significance level, there is a linear association between sales and test score

(f) $r^2 = .946286$ (coefficient of determination), $r = .9728$ 97.28% of the variability in y is explained by the regression model. This is quite high.

(g) $\hat{y} \pm t_{\alpha/2} S \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}}$ $17.4 \pm 5.841(1.3165) \sqrt{\frac{1}{5} + \frac{(15 - 14.4)^2}{91.2}}$

17.4 ± 3.47 (13.93, 20.87) in \$1000

(h) $\hat{y} \pm t_{\alpha/2} S \sqrt{\frac{1}{n} + 1 + \frac{(x^* - \bar{x})^2}{S_{xx}}}$ $17.4 \pm 5.841(1.3165) \sqrt{\frac{1}{5} + 1 + \frac{(15 - 14.4)^2}{91.2}}$

17.4 ± 8.44 (8.96, 25.84) in \$1000

8. Paired data

Wilcoxon Signed-Rank test

H_0 : the distributions are the same for dinner spending

H_a : the distributions are not the same for dinner spending

$W_+ = 23$ $W_- = 5$ (man- woman) $W = 5$

RHo if $W \leq 2$

Dec: Fail to RHo, $5 > 2$

Conclusion: At the 5% significance level, there appears to be no significant difference in spending for the males and females.

9. Kruskal Wallis test

H_0 : the distributions are the same for the 3 stores

H_a : distributions are not all the same for the 3 stores.

Rho if $KW > 5.991 (\chi^2_{.05}, 2)$

$$KW = \frac{12}{15(16)} \left(\frac{19.5^2}{5} + \frac{40.5^2}{5} + \frac{60^2}{5} \right) - 3(16) = 8.205$$

Dec: $RH_0, 8.205 > 5.991$

Conclusion: At the 5% significance level, there appears to be no significant difference in spending for the 3 stores..

10. (a) (i) $H_0: \mu \leq 1.5$

$H_a: \mu > 1.5$

$$Z = \frac{1.91 - 1.5}{2/\sqrt{100}} = 2.05 \quad P(z > 2.05) = 0.0202$$

$$(ii) Z = \frac{1.91 - 1.7}{2/\sqrt{100}} = 1.05 \quad P(z < 1.05) = 0.8531$$

$$(b)(i) n = \left(\frac{2.326(2)}{.233} \right)^2 = 398.6 \sim 399$$

$$(ii) 1.76 \pm 0.233 \quad (1.527, 1.993)$$

The CI is above 1.5 indicating that the advertisement is not true

11. (a) $H_0: p \leq .5$

$H_a: p > .5$

$$(b) (i) 1.645 = \frac{\hat{p} - .5}{\sqrt{(.5)(.5)/400}} \quad \hat{p} = .541125$$

$$\hat{p} = x/n \quad \hat{p}n = x \quad x = 400(.541125) = 216.45 \sim 217$$

$$(ii) \quad z \text{ calc} = 1 \quad P(z > 1) = .1587$$

12. (a) (i) $H_0: \mu_s \leq \mu_n$

$H_a: \mu_s > \mu_n$

RH_0 if $t_{calc} > 2.821$ ($df = 9, \alpha = .01$)

$$s_p^2 = 5.2775$$

$$t_{calc} = 10.42$$

Dec: $RH_0, 10.42 > 2.821$

Conclusion: At the 1% significance, the corrosion is less for the new paint

(ii) $H_0: \mu_s \leq \mu_n$

$H_a: \mu_s > \mu_n$

RH_0 if $t_{calc} > 3.365$ ($df = 5, \alpha = .01$)

$$t_{calc} = 9.739$$

Dec: $RH_0, 9.739 > 3.365$

Conclusion: At the 1% significance, the corrosion is less for the new paint

(b) H_0 : The corrosion is the same for both paints

Ha: The corrosion is higher for the old paint
 RHo if $M \geq 40$ $n_1=5, n_2=6$ $\alpha = .05$
 $M = 45$
 Dec: RHo, $45 > 40$
 Conclusion: Same as before

13. (a) $H_0: \mu \leq 3$
 $H_a: \mu > 3$
 RHo if $t_{calc} > 2.463$ ($df = 29, \alpha = .01$)
 $t_{calc} = 5.921$
 Dec: RHo, $5.912 > 2.463$
 Conclusion: At the 1% significance, the response time exceeds 3 seconds on average
- (b) $p\text{-value} = P(t > 5.921) = 0$ (computer)
 $p\text{-value} < .005$ (tables)
- (c) $H_0: \sigma \geq .5$
 $H_a: \sigma < .5$
 RHo if $\chi^2_{calc} < 17.7083$ ($df = 29, \alpha = .05$)
 $\chi^2_{calc} = 15.8804$
 Dec: RHo, $15.8804 < 17.7083$
 Conclusion: At the 5% significance, the standard deviation is lower than 0.5
- (d) $p\text{-value} = P(\chi^2 > 15.8804) = .0231$ (computer)
 $.01 < p\text{-value} < .025$

14. (a) $(.547 - .252) \pm 1.96 \sqrt{\frac{.547(.453)}{100} + \frac{.252(.748)}{100}}$ $.295 \pm .129$ (.166, .424)

- (b) Since zero does not fall in the 95% CI, this indicates that the % in 1982 is greater than the percentage now.
- (c) $H_0: p_{82} \leq p_{now}$
 $H_a: p_{82} > p_{now}$

15. Source	SS	df	MS	F
Trt	45.6	2	22.8	6
Block	60	4	15	3.947
error	30.4	8	3.78	

(test of blocks)
 H_0 : no difference between times
 H_a : Mean weights not the same for all times

Rho is $F_{block} > 3.838$ ($df = 4, 8, \alpha = .05$)
 $F_{block} = 3.947$
 Dec: RHo, $3.947 > 3.838$
 Conclusion: At the 5% significance level, mean weights are not the same for all times.

(test of treatments)

Ho: no difference between the processes

Ha: not all processes are the same

Rho if $F_{trt} > 4.459$ ($df = 2, 8$, $\alpha = .05$)

$F_{trt} = 6$

Dec: RHo, $6 > 4.459$

Conclusion: At the 5% significance level, not all processes are the same.

16. Ho: $\mu \leq 15$

Ha: $\mu > 15$

$Z_{calc} = 3.426$

p-value = $P(z > 3.426) = .0003$

Since p-value is small, we RHo and conclude that the advertisement is most likely false.

17. Ho: $\sigma \leq .95$

Ha: $\sigma > .95$

RHo if $\chi^2_{calc} > 18.307$ ($df = 10$, $\alpha = .05$)

$\chi^2_{calc} = 17.871$

Dec: Fail to RHo

Conclusion: At the 5% significance, the standard deviation is not greater than 0.95

18. (a) Ho: $\mu \leq 100,000$

Ha: $\mu > 100,000$

RHo if $t_{calc} > 1.753$ ($df = 15$, $\alpha = .05$)

$t_{calc} = 1.867$

Dec: RHo

Conclusion: At the 5% significance level, the firm's claim is false ($\mu > \$100,000$)

(b) p-value = $P(t > 1.867) = .0408$ (computer)

$.025 < \text{p-value} < .05$ (tables)

19. (a) Test for equal population variances.

Ho: $\sigma^2_1 = \sigma^2_2$

Ha: $\sigma^2_1 \neq \sigma^2_2$

RHo if $F_{calc} > 1.61$ (using 40 and 120 df because the table can't read 49 and 99 df)

Or $F_{calc} > 1.74$ (using 40 and 60 df because the table can't read 49 and 99 df)

$F_{calc} = 1.6^2 / .8^2 = 4$

Dec RHo

Conc: At the 5% significance level, the variances are not the same.

We do a non-pooled confidence interval

Df = 61 $t \sim 2$ (since it's close to 10 df) or we could use $z = 1.96$ since the sample size is large.

Using z:

$$(4.6 \pm 1.96 \sqrt{\frac{.8^2}{100} + \frac{1.6^2}{50}} \quad 4.6 \pm .47 \quad (4.13, 5.07)$$

$$4.6 \pm 2 \sqrt{\frac{.8^2}{100} + \frac{1.6^2}{50}} \quad 4.6 \pm .48 \quad (4.12, 5.08)$$

(b) Yes there is a difference because the CI does not include zero. The mean days for female is anywhere from 4.13 to 5.07 or (4.12 to 5.08) more than males.

20. (a)(i) $H_0: \mu \geq 507.5$

$H_a: \mu < 507.5$

(ii) $t_{\text{calc}} = -2.012$

(iii) $p\text{-value} = P(t < -2.012) = .0395$ (computer)

$.025 < p\text{-value} < .05$ (tables)

Since $p\text{-value}$ is small, the shop machine should be adjusted.

(b) Wilcoxon Signed Rank test

H_0 : The machine is working fine

H_a : The machine is not working fine (median < 507.5)

R H_0 if $W \leq 8$ ($n=9$ $\alpha = .05$)

$$W^+ = 8.5 \quad W^- = 36.5 \quad W = 8.5$$

Dec: Fail to R H_0 , $8.5 > 8$

Conclusion: At the 5% significance level, it appears that the machine should **not** be adjusted.

Sign test $S^+ = 3$ $S^- = 6$

$$P\text{-value} = P(X \geq 6) = 9C6(.5)^6(.5)^3 + 9C7(.5)^7(.5)^2 + 9C8(.5)^8(.5)^1 + 9C9(.5)^9(.5)^0 \\ = .2539$$

or

$$P(X \leq 3) = 9C3(.5)^3(.5)^6 + 9C2(.5)^2(.5)^7 + 9C1(.5)^1(.5)^8 + 9C0(.5)^0(.5)^9 \\ = .2539$$

Dec: Fail to R H_0 $.2539 > .05$