

## 14.2: Single-Population Inferences

- The **sign test** provides inferences about population *medians*, or *central tendencies*, when skewed data or an outlier would invalidate tests based on normal distributions.

## 14.2: Single-Population Inferences

- One-tailed test for a Population Median,  $\eta$
- Two-tailed test for a Population Median,  $\eta$

$$H_0: \eta = \eta_0$$

$$H_a: \eta > \eta_0 \text{ [or } H_a: \eta < \eta_0 \text{]}$$

$$H_0: \eta = \eta_0$$

$$H_a: \eta \neq \eta_0$$

Test Statistic:

*Test Statistic:*  
S = number of sample measurements greater than [less than]  $\eta_0$

S = larger of  $S_1$  and  $S_2$  where  $S_1$  is the number of measurements less than  $\eta_0$  and  $S_2$  the number greater than  $\eta_0$

## 14.2: Single-Population Inferences

- One-tailed test for a Population Median,  $\eta$   
*Observed significance level:*  
 $p\text{-value} = P(X \geq S)$
- Two-tailed test for a Population Median,  $\eta$   
*Observed significance level:*  
 $p\text{-value} = 2P(X \geq S)$

where X has a binomial distribution with parameters  $n$  and  $p = 0.5$ .

Reject  $H_0$  if the  $p$ -value is  $\leq \alpha$ .

## Starbucks Coffee

On 6 consecutive days, you purchased the same 16-ounce coffee from a Starbucks shop and determined the caffeine levels: 564 498 259 303 300 307 (ml)

Does the median caffeine level exceed 300 ml? (use alpha = 0.05 level)

You can test  $H_0: \eta = 300$  vs  $H_a: \eta > 300$ . Of the 6 measurements, one = 300 and we take it out. For the remaining 5, 4 are  $> 300$ . The  $p\text{-value} = P(X \geq 4 | n=5, p=0.5) = 1 - P(X \leq 3 | n=5, p=0.5) = 1 - 0.8125 = 0.1875 > 0.05$ , so you fail to reject the  $H_0$  at 5% level.

## 14.2: Single-Population Inferences

If  $n$  is large ( $n \geq 10$ ), the normal distribution table can be used. The test statistic is

$$z = \frac{(S - 0.5) - 0.5n}{0.5\sqrt{n}}$$

and we reject  $H_0$  for the one-tail test if  $z \geq z_{\alpha}$ , or reject  $H_0$  for the two-tail test if  $z \geq z_{\alpha/2}$ .

## 14.2: Single-Population Inferences

Median time to failure for a band of compact disc players is 5,250 hours. Twenty players from a competitor are tested, with failure times from 5 hours to 6,575 hours. Fourteen of the players exceed 5,250 hours.

Do the competitor's machines perform differently at  $\alpha = 10\%$  level?

## 14.2: Single-Population Inferences

$$H_0 : \eta = 5,250$$

$$H_a : \eta \neq 5,250$$

$$\alpha = .10$$

$$z_{\alpha/2} = z_{0.05} = 1.645$$

$$z = \frac{(S - .5) - 0.5n}{0.5\sqrt{n}} = \frac{13.5 - 10}{0.5\sqrt{20}} = 1.565 < 1.645$$

Do not reject  $H_0$

## 14.3: Comparing Two Populations: Independent Samples

### Wilcoxon Rank Sum Test

Used to test whether two independent samples have the same probability distribution

Given random sample 1,  $x_1, x_2, \dots, x_n$  and random sample 2,  $y_1, y_2, \dots, y_m$ , from distributions  $D_1$  and  $D_2$ , respectively. Sample 1 and sample 2 are independent of each other. Assign ranks 1 to  $n+m$  to the combined sample. Denote by  $T_1$  the sum of the ranks sample 1 receives and  $T_2$  the sum of the ranks sample 2 receives.

If there are ties, assign to each the average of the ranks that would have been assigned to them had there been no ties.

For example, if there are tied values from 5<sup>th</sup> to 8<sup>th</sup> position, assign  $6.5 = (5+6+7+8)/4$  to each of the four positions.

## 14.3: Comparing Two Populations: Independent Samples

### Wilcoxon Rank Sum Test

- One-tailed test
  - $H_0$ :  $D_1$  and  $D_2$  are identical
  - $H_a$ :  $D_1$  is shifted right of  $D_2$   
[or  $H_a$ :  $D_1$  is shifted left of  $D_2$ ]
  - Test statistic:
  - $T_1$ , if  $n_1 < n_2$
  - $T_2$ , if  $n_1 > n_2$
  - Either if  $n_1 = n_2$
  - Rejection region:
  - $T_1$ :  $T_1 \geq T_U$  [or  $T_1 \leq T_L$ ]
  - $T_2$ :  $T_2 \leq T_L$  [or  $T_2 \geq T_U$ ]
- Two-tailed test
  - $H_0$ :  $D_1$  and  $D_2$  are identical
  - $H_a$ :  $D_1$  is shifted either right or left of  $D_2$
  - Test statistic:
  - $T_1$ , if  $n_1 < n_2$
  - $T_2$ , if  $n_1 > n_2$
  - Either if  $n_1 = n_2$
  - Denote by  $T$  the statistic
  - Rejection region:
  - $T \leq T_L$  or  $T \geq T_U$

## Reaction Time Under Drug A & B

Drug A ( $n_1 = 6$ )		Drug B ( $n_2 = 7$ )	
Reaction Time	Rank	Reaction Time	Rank
1.96	4	2.11	6
2.24	7	2.41	8.5
1.71	2	2.07	5
2.41	8.5	2.71	11
1.62	1	2.50	10
1.93	3	2.84	12
		2.88	13

$$T_1 = 4+7+2+8.5+1+3 = 25.5$$

$$T_2 = 6+8.5+5+11+10+12+13 = 65.5$$

19

We want to test

$H_0$ :  $D_A$  and  $D_B$  are identical

$H_a$ :  $D_A$  is shifted right of  $D_B$  or  $H_a$ :  $D_A$  is shifted left of  $D_B$

Since  $6 = n_1 < n_2 = 7$ , our test statistic is  $T =$

$T_1$ . At  $\alpha = 0.05$  level, we reject  $H_0$  if

$T \leq T_L = 28$  or  $T \geq T_U = 56$ . Now  $T = 25.5 < 28$ ,

we reject  $H_0$  at 5% level to conclude that Drug B seems to increase the reaction time.

20

## 14.3: Comparing Two Populations: Independent Samples ( $n_1 \geq 10, n_2 \geq 10$ )

### Wilcoxon Rank Sum Test for Large Samples

- One-tailed test

$H_0$ :  $D_1$  and  $D_2$  are identical

$H_a$ :  $D_1$  is shifted right of  $D_2$

[or  $H_a$ :  $D_1$  is shifted left of  $D_2$ ]

Rejection region:

$$z \geq z_\alpha \text{ [or } z < -z_\alpha \text{]}$$

- Two-tailed test

$H_0$ :  $D_1$  and  $D_2$  are identical

$H_a$ :  $D_1$  is shifted either right

or left of  $D_2$

Rejection region:

$$|z| \geq z_{\alpha/2}$$

$$\text{Test Statistic: } z = \frac{T_1 - \frac{n_1(n_1 + n_2 + 1)}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$

21