STAT 327

Solutions to Assignment #3

6.2 (a) Here is a table showing the values of X = maximum number for rolls of two dice, for each of the 36 outcomes (with prob 1/36 each)

1st die \ 2nd die	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

So, e.g., $P[X=4] = P[(1,4),(2,4),(3,4),(4,1),(4,2),(4,3),(4,4)]$

$$= \sum_{i=1}^{7} \frac{1}{36} = \frac{7}{36}$$

So we obtain

x	$P[X=x]$
1	1/36
2	3/36
3	5/36
4	7/36
5	9/36
6	11/36

(b) Note that each $P[X=x] > 0$ and $\sum_x P[X=x] = \frac{1}{36}[1+3+5+7+9+11] = 1$.

6.2 Also find EX.

$$EX = \sum x\, P[X=x] = 1\cdot\tfrac{1}{36} + 2\cdot\tfrac{3}{36} + 3\cdot\tfrac{5}{36} + 4\cdot\tfrac{7}{36} + 5\cdot\tfrac{9}{36} + 6\cdot\tfrac{11}{36}$$

$$= \tfrac{1}{36}\left[1+6+15+28+45+66\right]$$

$$= \underline{\underline{\tfrac{161}{36}}} = \underline{\underline{4.4722}}$$

6.3 Let X = # of bases. From the description, the distibution of X is

x	$P[X=x]$
0	0.718
1	0.174
2	0.065
3	0.004
4	0.039

(a) $.718 + .174 + .065 + .004 + .039$

$= 1.000$ ✓

(b) $EX = \sum_x x\, P[X=x] = 0(.718) + 1(.174) + 2(.065) + 3(.004) + 4(.039)$

$= .174 + .130 + .012 + .156$

$= \underline{\underline{0.472}}$

(c) Since the mean can be interpreted as a long-term average of values of X, it does not have to be equal to one of the possible values of X.

6.4 (a) Let X = grade-point of a randomly selected student. Then, by the description, the distribution of X is:

x	$P[X=x]$
4	0.20
3	0.40
2	0.30
1	0.10
total	1.00

(b) $\mu = E(X) = 4(.20) + 3(.40) + 2(.30) + 1(.10)$

$= .8 + 1.2 + .6 + .1$

$= \underline{\underline{2.7}}$

Also, the variance of X is

$$\sigma^2 = E\left[(X-\mu)^2\right] = E(X^2) - \mu^2 = E(X^2) - (2.7)^2$$

$$= E(X^2) - 7.29$$

$$E[X^2] = \sum_x x^2 P[X=x] = 16(.2) + 9(.4) + 4(.3) + 1(.1)$$

$$= 3.2 + 3.6 + 1.2 + 0.1$$

$$= 8.1$$

so $\sigma^2 = 8.1 - 7.29 = 0.81$

Also the standard deviation of X is

$$\sigma = +\sqrt{\sigma^2} = \sqrt{0.81} = \underline{\underline{0.9}}$$

6.6 (a) Assume all 1000 picks are equally likely.

Then $P[\text{win } \$500] = \frac{1}{1000} = \underline{0.001}$

(b)

x	$P[X=x]$
0	0.999
500	0.001

(c) $\mu = E(X) = \sum x P[X=x] = 0(.999) + 500(.001)$

$= \underline{0.50} \leftarrow$ expected winnings is \$0.50 = 50¢

6.36 (a) Two types of outcomes (boy, girl) on each trial; On each trial, P[success] = P[girl] = .49; also it seems reasonable to suppose that the trials are independent (i.e., a previous child's sex has no bearing on the next child's sex).

(b) $n = 4$, $p = 0.49$

(c) $P[X=2] = \binom{4}{2}(.49)^2(.51)^2$

$= 6(.2401)(.2601) = \underline{0.3747}$

6.38 Let X = # of correct answers. The random guessing amounts to 4 indep. trials with prob $1/5 = 0.2$ of being correct on each trial.

(a) $P[X=4] = \binom{4}{4}(.2)^4(.8)^0 = (.2)^4 = \underline{0.0016}$

(b) $P[\text{pass}] = P[X \geq 3] = \binom{4}{3}(.2)^3(.8)^1 + .0016 = 4(.008)(.8) + .0016$

$= 0.0256 + .0016 = \underline{0.0272}$

6.49 (a) discrete — the r.v. has only 5 possible values.

(b) The 5 probabilities are positive and

$$\sum_{i=0}^{4} P(i) = .71 + .15 + .09 + .03 + .02 = 1.00 \checkmark$$

(c) $\mu = EX = \sum x\, p(x) = \cancel{0(.71)+1(.15)}$

$$= 0(.71) + 1(.15) + 2(.09) + 3(.03) + 4(.02)$$

$$= .15 + .18 + .09 + .08 = \underline{\underline{0.50}}$$

Also $\sigma^2 = Var(X) = E(X^2) - \mu^2$.

$$E(X^2) = \sum x^2 p(x) = 0(.71) + 1(.15) + 4(.09) + 9(.03) + 16(.02)$$

$$= .15 + .36 + .27 + .32 = 1.1$$

So $\sigma^2 = 1.1 - (0.5)^2 = 1.1 - 0.25 = \underline{\underline{0.85}}$

Alternative calculation:

$$\sigma^2 = E[(X-\mu)^2] = E[(X-0.5)^2] = \sum_x (x-0.5)^2 p(x)$$

$$= (0-0.5)^2(.71) + (1-0.5)^2(.15) + (2-0.5)^2(.09)$$
$$+ (3-0.5)^2(.03) + (4-0.5)^2(.02)$$

$$= (.25)(.71) + (.25)(.15) + (2.25)(.09) + 6.25(.03) + 12.25(.02)$$

$$= .1775 + .0375 + .2025 + .1875 + .245 = \underline{0.85}$$

(Note how much easier the other calculation of σ^2 is.)

6.68 Let X = # of brides (out of 4) who changed their names.
If the 4 were a random sample, then it would be reasonable to assume $X \sim \text{Binom}(n=4;\ p=0.8)$,
so $P[X=4] = \binom{4}{4}(.8)^4(.2)^0 = (.8)^4 = \underline{\underline{0.4096}}$

6.69 (a) Let X = # of wins. Under assumption of the binomial distribution, $n=6$, $p=0.5$
and $P[X=3] = \binom{6}{3}(.5)^3(1-.5)^{6-3}$
$$= \frac{(6)(5)(4)}{3(2)(1)}(.5)^6 = 20 \cdot \frac{1}{64} = \underline{\underline{0.3125}}$$

(b) The assumption is that the results of different games are independent, each with $P[\text{win}] = 1/2$.
<u>No</u> — these are not reasonable assumptions.
[The prob of beating a <u>bad</u> team is ~~[illegible]~~ greater than the prob of beating a <u>good</u> team].

6.72 (a) $X \sim \text{Binom}(n, p=10^{-6})$, so $EX = np = n \cdot 10^{-6}$

(b) $n \cdot 10^{-6} = 1 \Rightarrow n = 10^6$

~~[illegible]~~

(continued)

6.72 (continued)

(c) Let W = amount you win when you play once and Q = profit when you play once $= W - \$1$

Then W has distribution

w	$p(w)$
$\$10^5$	10^{-6}
$\$0$	$1 - 10^{-6}$

So $EW = \sum w\,p(w) = 10^5 \cdot 10^{-6} = \$0.10$

and $EQ = E[W - \$1] = E(W) - \$1 = -\$0.90.$

So if you play the lottery 10^6 times, then your total expected winnings is $10^6 \cdot 10^{-1} = 10^5$
$= \$100{,}000$

and your total expected profit is

$$10^6 \times [-\$0.90] = -\$900{,}000$$